REPRESENTATIONS OF GL_n OVER p-ADIC LOCAL FIELDS

London Number Theory Study Group, Spring 2021
Organised by Luis García and Sarah Zerbes

The goal of this study group is to learn some of the fundamental results about admissible representations of $\mathrm{GL}_n(F)$ where F is a p-adic local field. During the first part (8 talks) we will focus on understanding representations of $\mathrm{GL}_2(F)$. We will work towards understanding the statement of the Local Langlands Correspondence for GL_2 and the ingredients that go into its proof for odd p.

In the second part we will discuss the famous paper of Bernstein and Zelevinsky on representations of $\mathrm{GL}_n(F)$. The final talk will be a guest lecture by David Helm on some recent developments in the representation theory of p-adic groups.

Schedule of talks. The Study Group will meet Wednesdays, 2pm to 3.30pm, via Zoom. You can find the link in the weekly London-Number-Theorists Digest email.

(1) **Overview + organisational meeting** (13 January) **Luis García**
An introduction to admissible representations and the local Langlands correspondence for $\mathrm{GL}_2(F)$. Overview of some of the ideas that go into the proof.

(2) **Representations of locally profinite groups**
(20 January) **Xenia Dimitrakou**
Discuss some general theory of admissible representations of locally profinite groups. Briefly explain why $GL_n(F)$ is locally profinite when F is a p-adic local field. Define smooth, admissible and irreducible representations. Explain Schur's lemma and central character of irreducible representations. Define contragredient. Define induction and compact induction from a closed subgroup (including a discussion of the modular character). Proof of Frobenius reciprocity (just for induced reps, not for compactly induced). Explain the'no small subgroups' argument. If there is time: discuss Hecke algebras. Refs: [3, Chapter 1].

(3) **Parabolic induction** (27 January) **Andrew Graham**
Main goal is to understand the principal series of $GL_2(F)$. Define parabolically induced reps. Define Jacquet module and twisted Jacquet module. Prove main properties including exactness. State the main result on reducibility of principal series, and sketch the proof following Bushnell–Henniart. Definition of supercuspidal representations. Refs: [3, Chapter 3]

(4) **Representations of** $\mathrm{GL}_2(F)$ (3 February) **Louis Jaburi**
Discuss Hecke algebras, the unramified principal series and the Satake isomorphism (with explicit comparison to classical Hecke operators on modular forms). Explicit construction of supercuspidals using the Weil representation. Refs: [2, Chapter 4].

(5) **Whittaker and Kirillov models and the newvector**
(10 February) **Lambert A'Campo**

State and prove multiplicity one for Whittaker functionals. Discuss Whittaker and Kirillov models with examples. Also discuss the newvector. If there is time, explain the idea of proof of converse theorem using Kirillov model. Refs: [2, Chapter 4], [Jacquet–Langlands].

(6) **Local aspects of Tate's thesis** (17 February) **Ashwin Iyengar**
Describe the local theory for GL_1 follow Kudla's expository article [4]. Define and compute the local ζ function. Explain uniqueness principle underlying functional equation for ζ. Then introduce L and ϵ factors and local functional equation. Observe ϵ of twists determines the character. Refs: [4], [2].

(7) **L-functions and epsilon factors for** $GL(2)$ (24 February) TBD
Definitions following Bushnell-Henniart sections 24 and 26, and computations of some local L and epsilon values. Refs: [3].

(8) **Weil group and statement of LLC** (3 March) **Chris Birkbeck**
Structure of Weil group. Local class field theory and the Artin reciprocity map. Weil-Deligne representations. Explicit description for GL_2 and p odd: describe the possible values of Deligne's operator N, and sketch the proof that the irreducibles are dihedral. L and epsilon factors. Statement of the Local Langlands Correspondence. Refs: [3, Chapters 7 and 8].

(9) **Representations of** $\mathrm{GL}_n(F)$**. Bernstein–Zelevinsky classification. I** (10 March) **Johannes Girsch**
Discuss the main statements in Bernstein–Zelevinsky's paper. Give examples and explain what the statements look like in the Weil–Deligne side. Refs: [1], [5].

(10) **Representations of** $\mathrm{GL}_n(F)$**. Bernstein–Zelevinsky classification. II** (17 March) TBD
Give proofs for selected statements in Bernstein–Zelevinsky's paper. Refs: [1], [5].

(11) **Guest lecture** (24 March) **David Helm**

References

1. I. N. Bernšteĭn and A. V. Zelevinskiĭ, *Representations of the group* $GL(n, F)$*, where* F *is a local non-Archimedean field*, Uspehi Mat. Nauk **31** (1976), no. 3(189), 5–70. MR 0425030
2. Daniel Bump, *Automorphic forms and representations*, Cambridge Studies in Advanced Mathematics, vol. 55, Cambridge University Press, Cambridge, 1997. MR 1431508
3. Colin J. Bushnell and Guy Henniart, *The local Langlands conjecture for* GL(2), Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 335, Springer-Verlag, Berlin, 2006. MR 2234120
4. Stephen S. Kudla, *Tate's thesis*, An introduction to the Langlands program (Jerusalem, 2001), Birkhäuser Boston, Boston, MA, 2003, pp. 109–131. MR 1990377
5. François Rodier, *Représentations de* GL(n, k) *où* k *est un corps* p*-adique*, Bourbaki Seminar, Vol. 1981/1982, Astérisque, vol. 92, Soc. Math. France, Paris, 1982, pp. 201–218. MR 689531