Eisenstein Days at UCL

December 18-19, 2023

Titles and Abstracts

Hohto Bekki (Max Planck Institute for Mathematics)

Title: Conical zeta values and special values of partial zeta functions of totally real fields

Abstract: The conical zeta values are real numbers defined by certain multiple sums over cones, which can be seen as a generalization of the multiple zeta values. If the cones are rational, it is known that such conical zeta values are related to the cyclotomic multiple zeta values. On the other hand, it seems that little was known about the conical zeta values for non-rational cones. In this talk, I would like to explain a relation between such conical zeta values associated with certain algebraic non-rational cones and special values of partial zeta functions of totally real fields, by considering a certain "conical" Eisenstein series.

Nicolas Bergeron (ENS) (Talk delivered by Pierre Charollois)

Title: Elliptic units for complex cubic fields

Abstract: In the early 2000's Felder and Varchenko have defined a generalisation of the Euler gamma function that is associated to an elliptic curve. This remarkable *elliptic gamma function* is meromorphic in several variables and comes from mathematical physics. It satisfies modular functional equations under the group $\mathrm{SL}_3(\mathbf{Z})$ which make it an analogue of the Jacobi theta function. In this paper, we unveil the place that this function and its avatars play in number theory. Our main thesis is that these functions play the role of meromorphic modular functions in extending the theory of complex multiplication to complex cubic fields. In other words we propose a conjectural solution to Hilbert's 12th problem for complex cubic fields. We give a lot of numerical evidences that support this conjecture. In addition we prove an analogue of the Kronecker limit formula that expresses the logarithm of the modulus of these conjectural units as the derivative at $s = 0$ of a partial zeta function. This relates our conjectural units to the conjectural Stark units. Joint work with Pierre Charollois and Luis Garcia.

Henri Darmon (McGill University)

Title: Explicit class field theory and orthogonal groups. (Joint with Jan Vonk)

Abstract: Around 5 years ago Jan Vonk and I proposed a framework for defining singular moduli attached to real quadratic fields. I will present some simple concrete instances of a more general framework in settings where the base field is totally real, and explain how these special cases can be tackled.
These statements are inspired by the notion of rigid meromorphic cocycles for orthogonal groups of signature (r,r) described in joint work with Lennart Gehrmann and Mike Lipnowski, as well as by the calculations in Romain Branchereau's PhD thesis.

Lennart Gehrmann (Universität Duisburg–Essen)

Title: Rigid meromorphic cocycles for orthogonal groups

Abstract: I will talk about a generalization of Darmon and Vonk's notion of rigid meromorphic cocycles to the setting of orthogonal groups. These objects should be viewed as p-adic analogues of the meromorphic functions on orthogonal Shimura varieties with prescribed divisors constructed by Borcherds. After giving an overview over the general setting I will discuss the case of orthogonal groups attached to quadratic spaces of dimension 4 in more detail. In particular, I will highlight the similarities with the classical theory of Hilbert modular surfaces. This is an account of joint works with Henri Darmon and Michael Lipnowski, and with Xavier Guitart and Marc Masdeu.

Richard Hill (University College London)

Title: Construction of cocycles using Shintani functions

Abstract: I will describe an $(n-1)$-cocycle on $GL(n, \mathbb{Q})$, taking values in a certain space of distributions on $\mathbb{A}_f^n \setminus \{0\}$. Here $\mathbb{A}_f$ denotes the ring of finite adèles of $\mathbb{Q}$, and the distributions take values in the Laurent series $\mathbb{C}((z_1, ..., z_n))$. This cocycle can be used to evaluate special values of Artin L-functions on number fields at negative integers. The cocycle is based on an idea of Solomon, and is closely related to earlier work of Sczech.

Aleksander Horawa (University of Oxford)

Title: Automorphic forms and higher algebraic cycles

Abstract: We will explain a surprising relationship between the contributions to cohomology of automorphic forms and higher algebraic cycles. We will focus on three examples:

(1) Hilbert modular forms: we conjecture that the cohomology associated with weight one Hilbert modular forms is related to Stark units for the associated Artin representation.

(2) Bianchi modular forms: Kartik Prasanna and Akshay Venkatesh conjecturally relate the cohomology certain arithmetic groups to higher algebraic cycles on a product of elliptic curves over imaginary quadratic fields.

(3) Siegel modular forms: in recent work with Kartik Prasanna, we conjecture that the cohomology associated with weight two Siegel modular forms is related to higher algebraic cycles on a product of abelian surfaces over Q. We will discuss some evidence to support these conjectures, and we hope that the recent advances towards Stark's conjecture and in the theory of Eisenstein cycles can shed light more light on them.

Alice Pozzi (University of Bristol)

Title: Generating Series for values of rigid meromorphic cocycles

Abstract: Rigid meromorphic cocycles are cocycles for some p-arithmetic groups acting on p-adic symmetric spaces. Their values at special points are conjectured to belong to class fields of some suitable global fields. In previous work with Darmon and Vonk, we proved a very special instance of this conjecture, exploiting, among other tools, the spectral decomposition of a generating series for the values of rigid meromorphic cocycles.

In this talk, we discuss similar generating series in a general framework involving biquadratic extensions. This is joint work in preparation with Judith Ludwig, Isabella Negrini, Sandra Rozensztajn and Hanneke Wiersema.

Robert Sczech (Rutgers University)

Title: An Eisenstein Cocycle For GL_2 With One Infinite Place

Abstract: The Eisenstein cocycle for GL_n can be viewed as a generalization of a Bernoulli polynomial ($n = 1$) to totally real number fields ($n \geq 1$) which parametrizes all special values of zeta and L-functions subject to the theorem of Klingen-Siegel. In our talk, we incorporate one infinite place into the Eisenstein cocycle in the case $n = 2$. The resulting cocycle parametrizes all those special values of L-functions in real quadratic number fields which are subject of the Stark conjecture in the abelian rank one case.

Jan Vonk (Universiteit Leiden)

Title: p-adic height pairings of geodesics

Abstract: I will discuss some recent progress on a tentative theory of differences of singular moduli for real quadratic fields, focussing on the role played by a certain p-adic height pairing of real quadratic geodesics on modular curves.

Chris Williams (University of Nottingham)

Title: p-adic L-functions for GL(3)

Abstract: Let π be a p-ordinary cohomological cuspidal automorphic representation of $\mathrm{GL}(n, \mathbb{A}_{\mathbb{Q}})$. Coates and Perrin-Riou conjectured that the (algebraic parts of) special values of its L-function satisfy beautiful p-adic congruences, captured in the existence of a p-adic L-function. For $n = 1, 2$ constructions of such p-adic L-functions predate the conjecture by decades, but for $n > 2$ our understanding is quite incomplete: in all previous constructions, π is assumed to be (at least) a functorial transfer via a proper subgroup of GL(n) (e.g. for symmetric squares from GL(2), Rankin–Selberg transfers, π admitting Shalika models, etc).

In this talk, I will describe joint work with David Loeffler, where we construct a p-adic L-function when $n = 3$, without any transfer/self-duality assumptions. Our method uses a 'Betti Euler system', where we construct a tower of integral Eisenstein classes in the Betti cohomology for GL(3).