

# The D'Hondt Method Explained

Helen J. Wilson, Mathematics Department, UCL

## Abstract

The D'Hondt method will be used in the European elections on May 22, 2014. Its description in the media traditionally focusses on the **process**: that is, the complex, algorithmic detail of how seats are allocated in order.

In this short article I provide a clearer explanation of the final allocation of seats and show that the two forms are equivalent.

Incidentally, the Jefferson method (used to distribute seats in the US House of Representatives among states) is algorithmically different but also comes to the same allocation of seats.

## Seat Allocation

Suppose you are in charge of allocating seats to parties. Once the votes have all been cast and counted, you are faced with a group of parties each of which has a certain number of votes.

You have a set of seats to allocate. The D'Hondt method is, in principle, very simple. You simply "sell" each seat to a party. Each seat "costs" the same number of votes; and each party buys as many seats as it can (and its leftover votes that are worth less than a seat, are discarded).

Your job is to set the price of a seat so that, at the end of this process,

- there are no seats left unsold; and
- no party has enough votes left over to buy another seat.

The technical method is simply a process whereby you initially set the price too high, and iteratively reduce it until you reach the right value.

## Example

Consider a set of 230,000 votes case for 4 parties:

Party	Votes	Price of seat (000s of votes)				
		32	30	26	25	20
A	100,000	3	3	3	4	5
B	80,000	2	2	3	3	4
C	30,000	0	1	1	1	1
D	20,000	0	0	0	0	1
Total seats		5	6	7	8	11

We will suppose that 7 seats are available<sup>1</sup>

<sup>1</sup>This data set is partly based on one taken from [http://en.wikipedia.org/wiki/D'Hondt\\_method](http://en.wikipedia.org/wiki/D'Hondt_method)

**Price too high** Set the price at 30,000 votes per seat. Party A can buy 3 seats (total cost 90,000, with 10,000 spare votes); party B only 2 seats (with 20,000 spare votes); party C just manages to buy a single seat; and party D cannot afford a seat. You have sold only 6 of your 7 seats.

**Price too low** Try a price of 25,000 votes per seat. Party A buys exactly 4 seats, party B 3 (with 5,000 spare votes), party C 1 (also with 5,000 spare votes) and party D is still out of the market. You have sold 8 seats, which is too many.

For this example, any price between 25,001–26,666 votes per seat will produce the desired outcome, in which the 7 seats are allocated A:3, B:3, C:1, D:0.

## How it works

The D'Hondt method consists of an iterative process, where each party is given a number

$$N = \frac{V}{s + 1}$$

where  $V$  is its total number of votes, and  $s$  is the number of seats it has already been allocated. At each stage, the party with the highest value of  $N$  is given a seat, and its value of  $s$  goes up by one.

The number  $N$  represents the highest price that party could pay for its next seat. So if it has no seats yet, it could pay all its votes if necessary. But if it has a seat already, it will need to pay for two seats if it is to make progress, so it can only afford  $V/2$  per seat from now on. And if it has  $s$  seats already, it will only be able to afford another if the price comes down to  $V/(s + 1)$ .

So as *price-setter* you start with the highest possible price, and sequentially lower the price until all your seats are sold. Initially you simply allocate a seat to the party that received most votes. But as time goes on, you ask each party how much it could pay to have one more than its current allocation (this will be its number  $N$ ), and allocate the next seat to the party with the highest offer.

Once you've "sold" all your seats, the process is complete.