Beyond the Hodge Theorem: curl and asymmetric pseudodifferential projections

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Consider the operator curl $:= *d$ acting on 1-forms over a connected oriented closed Riemannian 3-manifold. Put $P_\pm := \theta(\pm\,\mathrm{curl})$, θ being the Heaviside step function. The operators $P_\pm$ are completely determined by the Riemannian manifold and its orientation, and they constitute an orthonormal pair of projections which decompose the Hilbert space of real-valued coexact 1-forms into two orthogonal subspaces. We prove that the operators $P_\pm$ are pseudodifferential, write down their principal and subprincipal symbols and provide an algorithm for the explicit computation of their full symbols. We then consider the operator $P_+ - P_-$ and take its pointwise matrix trace. This gives us a scalar pseudodifferential operator A which we call *the asymmetry operator*. We prove that A is an operator of order -3 and define its regularised operator trace. This trace is a differential geometric invariant, a measure of the asymmetry of our Riemannian manifold under change of orientation.

The talk is based on the use of pseudodifferential techniques developed in a series of recent joint papers by Matteo Capoferri (Cardiff) and myself.