LOCALISATION OF AN IMPULSIVE ACOUSTIC SOURCE IN AN URBAN ENVIRONMENT

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Abstract

This paper concerns the inverse problem of localising an impulsive acoustic source in an urban environment using measurements from multiple receivers (microphones) distributed through the propagation domain. In free space the inverse problem is readily solved by Time Difference Of Arrival (TDOA) localisation. In urban environments there may not be a line-of-sight (LOS) between the source and each receiver, and an application of the free space TDOA method may produce erroneous results, especially if the streets are narrow. In this work the case of a network of 2D streets intersecting at right angles is considered, and an alternative TDOA method is proposed which explicitly takes into account the non-line-of-sight (NLOS) propagation. The results of a numerical simulation are also presented, together with comparisons with the free space method. The applicability of the new method to the 3D case is also discussed.

INTRODUCTION

The main analytical and numerical difficulties in the study of urban sound propagation are due to geometric complexity. The presence of buildings and other obstacles gives rise to complicated scattering effects, and introduces multiple propagation paths between source and receiver. This makes it difficult for a listener to locate the source of the sound, particularly when the source is not in a direct line-of-sight (LOS). This in turn has implications for situational awareness and event localisation, where the goal is to obtain an estimate of the source location, using the minimum possible number of receivers and in near real-time.

One approach to the localisation problem for an impulsive source, proposed in [1], is that of time-reversal processing. Although numerical simulations and field trials have shown this approach to be remarkably effective in determining the source location, even in fairly general urban environments, the method is currently too computationally intensive to be used in practical applications.

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The approach we consider in this paper is that of Time Difference Of Arrival (TDOA) localisation. The standard free space TDOA method is widely employed in the context of radio wave propagation in Global Positioning Systems (GPS) and mobile telephone handset localisation (see e.g. [2], [3]). In this paper we consider the application of TDOA techniques to acoustic wave propagation in urban environments. For the most part we restrict our attention to 2D environments, but some remarks will be made about 3D environments in the Appendix.

We remark that the TDOA method requires knowledge simply of the arrival time of the pulse at each receiver, and does not make use of additional information such as the amplitude of the pulse or the arrival direction. The question of how such additional information could be incorporated into the localisation method will not be considered here.

TDOA LOCALISATION IN FREE SPACE

In this section we briefly review the TDOA localisation method in 2D and 3D free space.

We suppose that a single pulse is emitted from an unknown source location $\mathbf{x_0}$ at an unknown time t_0, with the medium having been at rest for $t < t_0$. Assuming that the propagation speed c is constant throughout the medium, the arrival times t_i of the pulse at the receiver locations $\mathbf{x_i}$, $i = 1, ..., N$ are related to the Euclidean distance $|\mathbf{x_0} - \mathbf{x_i}|$ between the source and each receiver by the formula

$$c(t_i - t_0) = |\mathbf{x_0} - \mathbf{x_i}|. \tag{1}$$

Since the emission time t_0 is unknown, a single receiver provides no information about the source location. However, for any pair of receivers $\{i, j\}$, the range difference $D_{ij} = |\mathbf{x_0} - \mathbf{x_i}| - |\mathbf{x_0} - \mathbf{x_j}|$ can be computed from the measured TDOA $t_i - t_j$. Knowledge of D_{ij} places the source on a particular branch of a hyperbola (or hyperboloid of revolution in 3D) with foci at $\mathbf{x_i}$ and $\mathbf{x_j}$. The localisation problem with N receivers is therefore reduced to the problem of determining the mutual intersection point of a set of $\binom{N}{2}$ hyperbolae (hyperboloids of revolution in 3D) that result from considering every possible distinct pair of receivers.

With $N \geq 4$ ($N \geq 5$ in 3D), the source location can be reduced to the solution of a simple linear equation [4]. This equation can be shown to have a unique solution under certain non-degeneracy assumptions (including the obvious constraint that the receivers are not colinear (or coplanar in 3D)), although the details are not presented here.

TDOA LOCALISATION IN AN URBAN ENVIRONMENT

In an urban environment, a single pulse emitted from the source may give rise to multiple pulse arrivals at a receiver, due to the effects of reflection and diffraction by buildings. In this case, equation (1) is replaced by

$$c(t_i - t_0) = L_i, \tag{2}$$

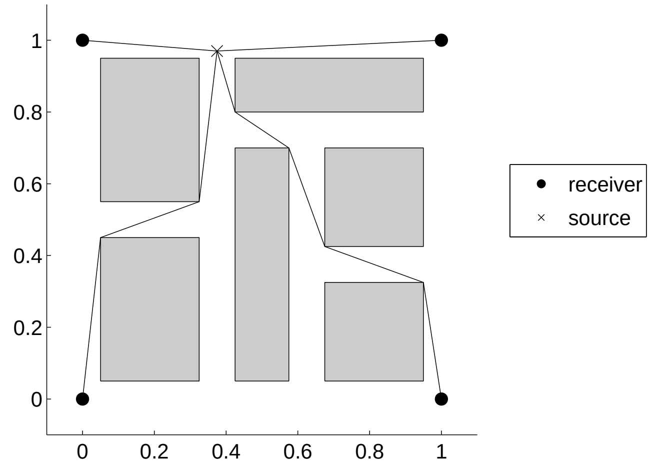


Figure 1: *Shortest ray paths from source to receivers in a 2D urban environment*

where t_i is the arrival time of the *first* pulse to reach the receiver, and L_i is the length of the *shortest* ray path between the source and receiver. This path is either a LOS path or a (multiply-) diffracted path, as illustrated in Figure 1. The lack of a simple analytical expression for L_i in such a domain means that recovering the source location exactly from the range differences $D_{ij} = L_i - L_j$ is a difficult task in general.

If the shortest ray path between source and receiver does not deviate too much from the free space path (for example, if the buildings are sparsely distributed, or, in 3D, if the buildings are low), we may approximate

$$L_i \approx |\mathbf{x_0} - \mathbf{x_i}|, \tag{3}$$

and an estimate of the source location can be obtained by an application of the free space solution to the recorded TDOA data. However, the TDOA data will, in general, be inconsistent, in the sense that the $\binom{N}{2}$ hyperbolae (hyperboloids of revolution in 3D) no longer have a point of mutual intersection. A number of methods have been proposed for estimating the source location in this case (see e.g. [3], [5]), all of which involve solving the TDOA equations in some approximate sense. In this paper we adopt the approach taken in [3], seeking an estimated source location $\hat{\mathbf{x}}_\mathbf{0}$ which minimises the function

$$F(\mathbf{x_0}, t_0) := \sum_{i=1}^{N} \left(c(t_i - t_0) - |\mathbf{x_0} - \mathbf{x_i}|\right)^2. \tag{4}$$

In the case of tall, densely distributed buildings, however, we cannot expect such an approach to perform well in general. In the next section we consider a particular type of urban environment in which an alternative approximation for L_i is available, which, as we shall see, leads to a more accurate estimate of the source location than that provided by the minimiser of (4).

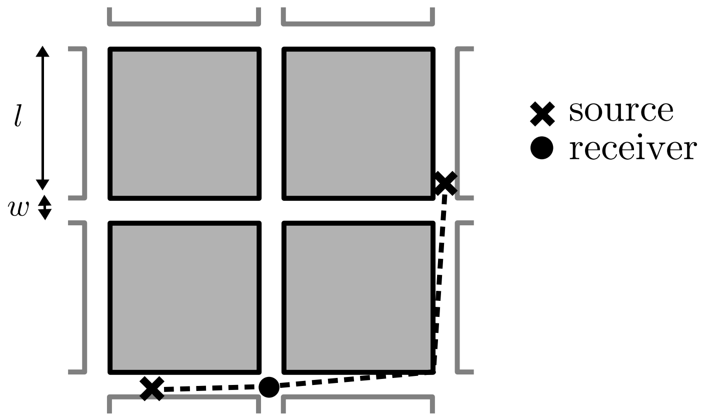


Figure 2: *A network of 2D streets intersecting in right-angled crossroads*

A NETWORK OF NARROW STREETS IN 2D

We consider first a network of 2D streets intersecting at right-angled crossroads, as illustrated in Figure 2. After nondimensionalising lengths by a typical receiver separation d, which we assume is of the order of the typical street length l, we consider the regime in which the (nondimensional) street width $w \ll 1$. We then propose to approximate the length of the shortest ray path by

$$L_i \approx |\mathbf{x_0} - \mathbf{x_i}|_{\text{sum}} \left(1 - O\left(w\right)\right), \qquad w \ll 1, \tag{5}$$

where

$$|\mathbf{x_0} - \mathbf{x_i}|_{\text{sum}} = |x_0 - x_i| + |y_0 - y_i| \tag{6}$$

is the 2D sum norm and the minus sign in (5) indicates that the approximation is an *overestimate*. The approximation (5) will be valid provided that $|\mathbf{x_0} - \mathbf{x_i}|_{\text{sum}} \gtrsim O\left(1\right)$ and the receiver $\mathbf{x_i}$ is located at a junction.

Under the approximation (5), knowledge of the range difference D_{ij} for any pair of receivers places the source location at points

$$H_{ij} := \left\{\mathbf{x} \,:\, |\mathbf{x} - \mathbf{x_i}|_{\text{sum}} - |\mathbf{x} - \mathbf{x_j}|_{\text{sum}} = D_{ij}\right\}, \tag{7}$$

analogous to the hyperbolae of the 2D free space case. With N receivers, the source location lies within the mutual intersection of the $\binom{N}{2}$ such H_{ij} that result from considering every possible distinct pair of receivers.

We immediately notice that if we define R to be the smallest rectangle containing the N receivers, as illustrated in Figure 3, then no source location outside of R can be uniquely determined from the TDOA data. For a source located in region I of Figure 3, we can at best hope to determine the x coordinate of the source location, and whether the source lies ‘North’ or ‘South’ of the rectangle R. Similarly, for a source located in region II, we can at best hope to determine the y coordinate of the source location, and whether the source lies ‘East’ or ‘West’ of the rectangle R.

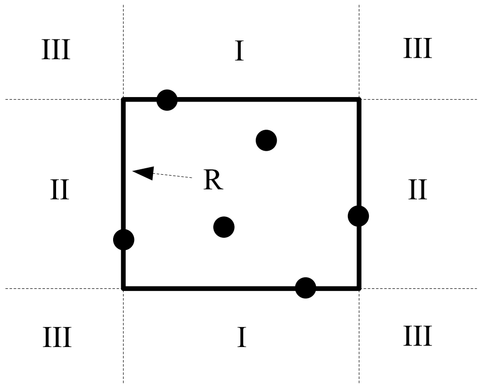
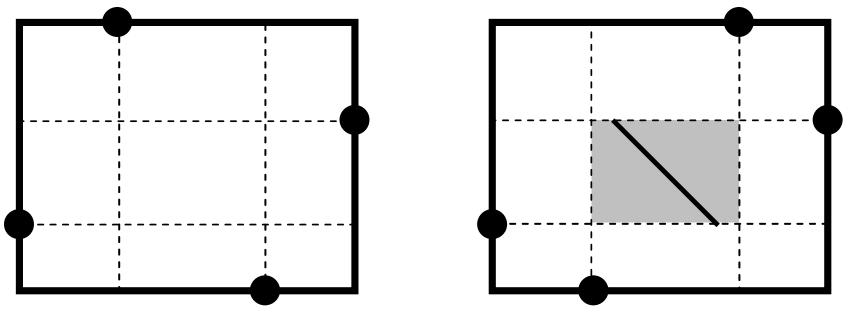


Figure 3: *Regions (I-III) in which a source cannot be located uniquely in the sum-norm approximation*

For a source in region III, we can at best hope to determine in which of the four quarter planes the source lies.

Thus, for the purposes of this paper, we restrict our attention to source locations lying inside the rectangle R. Even then, the question of which receiver configurations admit a unique solution to the TDOA problem has yet to be resolved. For example, of the two four-receiver configurations illustrated in Figure 4, the configuration in Figure 4(a) does admit a unique solution to the TDOA problem, whereas that in Figure 4(b) does not, since the TDOA data for a source located inside the shaded rectangle will be the same as that for any source location on a diagonal line segment passing through the source.

In the special case where $x_i = x_j$ or $y_i = y_j$, the set H_{ij} takes a particularly simple form. For example, consider the case where $y_i = y_j$ and $x_i < x_j$. Then for $|D_{ij}| < x_j - x_i$, H_{ij} is the straight



(a) (b)

Figure 4: *TDOA localisation in the sum-norm approximation. The TDOA problem for the four-receiver configuration in (a) admits a unique solution for any source location inside the outer rectangle. In contrast, for the four-receiver configuration in (b), source locations inside the inner (shaded) rectangle cannot be recovered uniquely from the TDOA data. In particular, any source location on the diagonal line segment pictured will produce the same TDOA data between the 4 receivers.*

line

$$x = \frac{1}{2}\left(D_{ij} + (x_j - x_i)\right). \tag{8}$$

When $|D_{ij}| = x_j - x_i$, H_{ij} is the half-plane $x > x_j$ (if $D_{ij} > 0$) or the half-plane $x < x_i$ (if $D_{ij} < 0$). Finally, if $|D_{ij}| > x_j - x_i$, H_{ij} is empty. Similarly, when $x_i = x_j$ and $y_i < y_j$, the same statements apply, but with x's switched with y's throughout.

The implication of these observations is that if the source is known to lie in a given rectangular subregion R, the source location can be recovered uniquely from the TDOA data from just three receivers, placed at any three of the vertices of R. The pair of receivers with the same y coordinate can be used to determine the x-coordinate of the source, by formula (8), and the pair of receivers with the same x coordinate can be used to determine the y-coordinate of the source similarly.

More generally, in the case of inconsistent TDOA data and with N receivers, we may seek, as in (4), an estimated source location $\hat{\mathbf{x}}_\mathbf{0}$ which minimises

$$F_{\text{sum}}(\mathbf{x_0}, t_0) := \sum_{i=1}^{N} \left(c(t_i - t_0) - |\mathbf{x_0} - \mathbf{x_i}|_{\text{sum}}\right)^2. \tag{9}$$

We now present an illustration of this.

RESULTS OF 2D NUMERICAL SIMULATION

The minimisations (4) and (9) have been compared in the 2D urban environments illustrated in Figure 5. These environments are more general than the network of streets intersecting in cross-roads considered in the previous section, but the choice of receiver locations ensures that (5) will nonetheless hold when $w \ll 1$.

In the numerical simulation, the shortest propagation paths between a number of different source locations and each receiver have been computed by a shortest-path algorithm, a typical output of which is shown in Figure 1. For each source location, the arrival times at each receiver were computed, assuming a propagation speed $c = 1$, and taken as input data to the two minimisations (4) and (9). Initially, the only constraint to the minimisation was that the solution should be located within the rectangle R. If the resulting solution was found to lie inside a building, a second minimisation over the walls of that building was performed to obtain the final estimate.

In both steps the minimisation was carried out in Matlab using the built-in nonlinear least-squares routine `lsqnonlin`, which is based on an iterative trust-region-reflective algorithm. The initial guess for the iteration was taken to be

$$(x_0, y_0, t_0)^{\text{initial}} = \left(1/2, 0, \min_i \{t_i\} - 1/2\right), \tag{10}$$

but the results were found to agree for five different choices of initial guess.

The results of the calculation are plotted graphically in Figure 5, and the maximum and mean localisation errors (measured in terms of Euclidean distance) are displayed in Table 1.

Table 1: *Results of 2D numerical simulation.*

Street width	Euclidean minimisation		Sum-norm minimisation	
	max error	mean error	max error	mean error
(a) $w = 0.2$	0.067	0.027	0.184	0.127
(b) $w = 0.1$	0.127	0.061	0.147	0.084
(c) $w = 0.025$	0.135	0.084	0.027	0.019

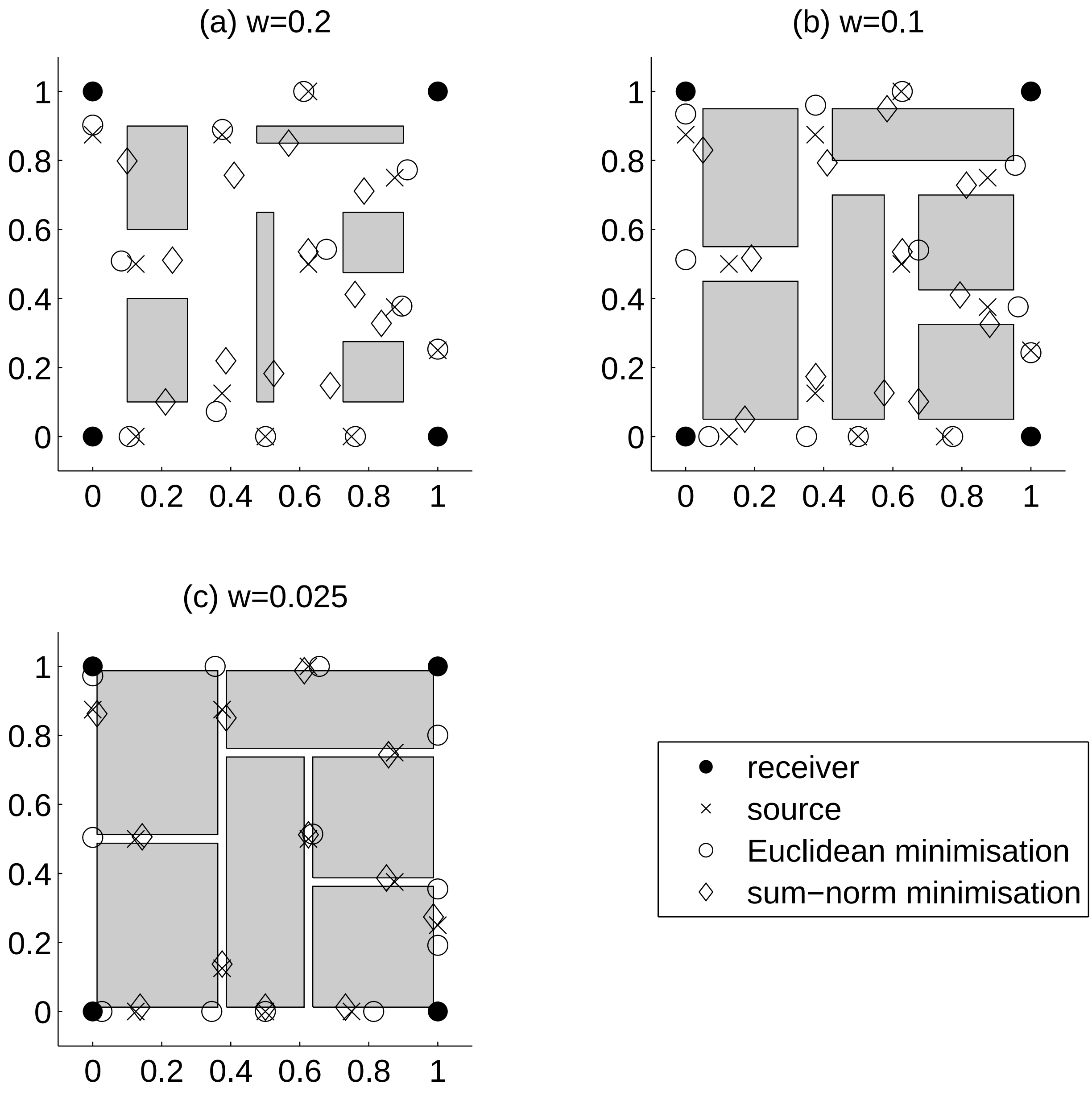
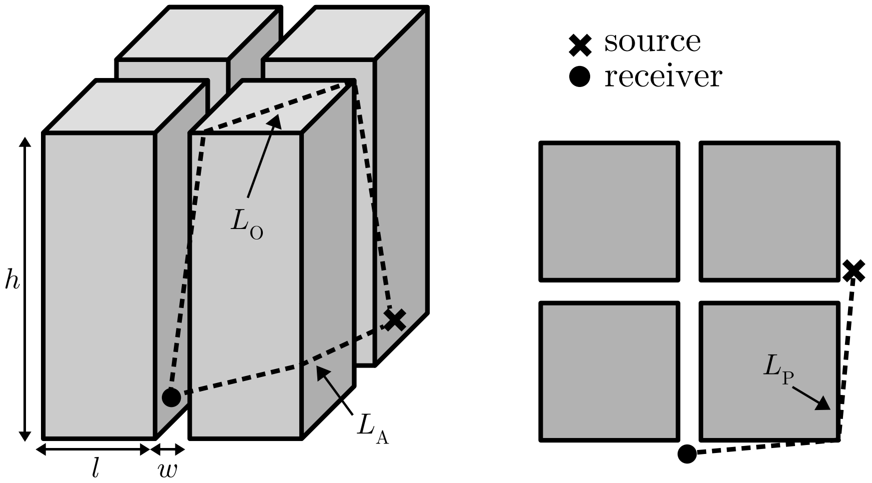


Figure 5: *Results of 2D numerical simulation.*



(a) 3D environment (b) Projection onto xy-plane

Figure 6: *Diffracted ray paths in a 3D urban environment*

The domain in Figure 5(a) represents a relatively sparse urban environment. In this domain the Euclidean minimisation (4) performs remarkably well, given that many of the source locations considered have no LOS to any of the four receivers. The domain in Figure 5(c) represents a relatively dense urban environment, and here the sum-norm minimisation (9) provides a more accurate estimate than the Euclidean minimisation. The domain in Figure 5(b) represents an intermediate case, in which the performance of the two minimisations is comparable.

CONCLUSION AND OUTLOOK

In this study a new TDOA method has been proposed for the localisation of impulsive acoustic sources in a network of 2D streets intersecting at right angles. In the case of narrow streets it has been shown, by means of a numerical simulation, to provide a more accurate estimate of the source location than the standard free space TDOA method.

APPENDIX: APPLICABILITY TO THE 3D CASE

In this Appendix we discuss the conditions under which the new 2D TDOA method proposed in this paper can be applied to the case of a network of narrow streets intersecting in right-angled crossroads in 3D, as illustrated in Figure 6(a).

The first requirement is that the buildings are tall enough to ensure that the shortest ray path between source and receiver involves only diffraction *around* buildings and not *over* them. In order to derive a sufficient condition for this to occur, we note that for a source and receiver below

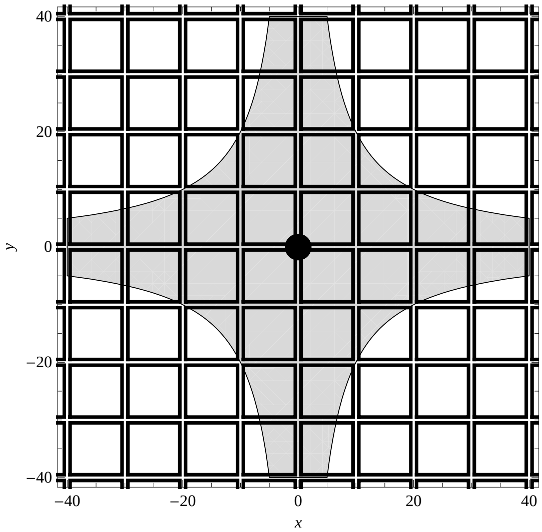


Figure 7: *Illustration of the region (shaded) in which the condition (14) holds. The receiver is located at the origin and the (constant) building height, street length and street width are* $h = 10$, $l = 9$, $w = 1$, *respectively. Both the source and receiver are assumed to be at ground level. The bounding curves of the shaded region are hyperbolae with centre at the receiver.*

building height, the length L_A of the shortest ray path around buildings is given by

$$L_{\mathrm{A}} = \sqrt{L_{\mathrm{P}}^2 + (z_0 - z_i)^2}, \tag{11}$$

where L_{P} is the length of the projection of the ray path onto the xy-plane (see Figure 6(b)). Provided that the receiver $\mathbf{x_i}$ is located at a junction we have

$$L_{\mathrm{A}} \leq \sqrt{(|x_0 - x_i| + |y_0 - y_i|)^2 + (z_0 - z_i)^2}. \tag{12}$$

Assuming a typical building height h, the length L_{O} of the shortest ray path over buildings can be bounded below by the length of the shortest piecewise-linear curve in 3D free space from the source to the receiver which reaches a height h, i.e.

$$L_{\mathrm{O}} \geq \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (h - z_0 + h - z_i)^2}. \tag{13}$$

By combining (12) and (13), we certainly have $L_{\mathrm{A}} \leq L_{\mathrm{O}}$ whenever

$$|x_0 - x_i||y_0 - y_i| \leq 2(h - z_0)(h - z_i). \tag{14}$$

An illustration of the region described by (14) is given in Figure 7. In practice, we expect the region in which $L_{\mathrm{A}} \leq L_{\mathrm{O}}$ to be larger than that described by (14), since the lower bound (13) is, in general, rather crude. However, a tighter lower bound for L_{O} has not yet been found.

Assuming that $L_{\mathrm{A}} \leq L_{\mathrm{O}}$, we proceed to nondimensionalise lengths by a typical receiver separation d, which we assume is of the order of the typical street length l. When the typical (nondimensional) street width $w \ll 1$ we approximate

$$L_{\mathrm{A}} \approx \sqrt{(|\mathbf{x_0} - \mathbf{x_i}|_{\mathrm{sum}})^2 + (z_0 - z_i)^2}\,(1 - O(w))\,, \tag{15}$$

which is valid provided that $|\mathbf{x_0} - \mathbf{x_i}|_{\text{sum}} \gtrsim O(1)$ and the receiver $\mathbf{x_i}$ is located at a junction. If the source and receiver are at similar heights, with $|z_i - z_0| \ll |\mathbf{x_0} - \mathbf{x_i}|_{\text{sum}}$, we can further approximate (15) to obtain

$$L_{\text{A}} \approx |\mathbf{x_0} - \mathbf{x_i}|_{\text{sum}} \left(1 - O(w) + O\left(\frac{|z_i - z_0|^2}{|\mathbf{x_0} - \mathbf{x_i}|_{\text{sum}}^2}\right)\right), \tag{16}$$

and the new 2D TDOA method can be applied to obtain an estimate of the source location.

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