MATHEMATICS 1302 (Applied Mathematics 2)
YEAR 2008–2009, TERM 2

HANDOUT #1: NEWTON'S FIRST LAW AND THE PRINCIPLE OF RELATIVITY

Newton's First Law of Motion

Our experience seems to teach us that the "natural" state of all objects is at rest (i.e. zero velocity), and that objects move (i.e. have a nonzero velocity) only when forces are being exerted on them. Aristotle (384 BC – 322 BC) thought so, and many (but not all) philosophers and scientists agreed with him for nearly two thousand years.

It was Galileo Galilei (1564–1642) who first realized, through a combination of experimentation and theoretical reflection, that our everyday belief is *utterly wrong*: it is an illusion caused by the fact that we live in a world dominated by friction. By using lubricants to reduce friction to smaller and smaller values, Galileo showed experimentally that objects tend to maintain nearly their initial velocity — whatever that velocity may be — for longer and longer times. He then guessed that, in the idealized situation in which friction is *completely* eliminated, an object would move *forever* at whatever velocity it initially had. Thus, an object initially at rest would stay at rest, but an object initially moving at 100 m/s east (for example) would continue moving forever at 100 m/s east. In other words, Galileo guessed:

> An isolated object (i.e. one subject to no forces from other objects) moves at constant velocity, i.e. in a straight line at constant speed. Any constant velocity is as good as any other.

This principle was later incorporated in the physical theory of Isaac Newton (1642–1727); it is nowadays known as **Newton's first law of motion**.

The Principle of Relativity

Newton's first law, though seemingly simple, has all sorts of subtleties hidden within it. One of them has to do with our choice of coordinate system — or, in physicists' language, with our choice of **frame of reference**.

The problem arises first at the level of kinematics (i.e. description of motion). Here's an example:

> Today I took the train from London to Edinburgh. I had breakfast in the dining car of the train, and a few hours later I had lunch in the dining car of the train. Did I have breakfast and lunch in the *same place*?

With respect to the *earth* frame of reference, the answer is *no*: I had breakfast in London and lunch in Edinburgh. But with respect to the *train* frame of reference, the answer is *yes*:

I had both breakfast and lunch in the dining car. Clearly, whether two events occurred in the same place or in different places depends on what frame of reference is being used. The question "Did breakfast and lunch occur in the same place?" makes sense only once we have agreed on a choice of frame of reference.

For *describing* motion, any frame of reference is as good as any other. Not so for the "laws of Nature" that specify *in what way* objects move! In particular, not so for Newton's First Law! Suppose, for example, that while you're in a lab observing a cart move at constant velocity along an air track, someone zooms by in a car that is accelerating north at 2 m/sec^2. That person in the car will *not* see your cart move at constant velocity; she will see it *accelerate south* at 2 m/sec^2. Or to take another example, an observer rotating on a merry-go-round located next to your lab will not see your cart move at constant velocity (i.e. in a straight line at constant speed); rather, he will observe its path to *curve*.

So, if you want to use the usual laws of physics — in particular, if you want Newton's First Law to hold — you cannot use any old frame of reference. Newton's First Law holds only with respect to certain very special frames of reference: these are called **inertial frames of reference**. An inertial frame is, by definition, one in which isolated objects move at constant velocity, i.e. one with respect to which Newton's First Law holds. So, Newton's First Law is in part just the *definition* of "inertial frame of reference"; but it is also the highly nontrivial empirical statement that *inertial frames of reference exist*. (To a good approximation, a frame of reference attached to the earth is inertial. But it's not exactly inertial, due to the rotation of the earth as well as to the earth's motion around the sun.)

I said "inertial frame*s* of reference exist", plural, implying that there is more than one such frame. And indeed that's so: if I have one inertial frame of reference, then any other frame of reference that is *moving at constant velocity* and *nonrotating* with respect to the first frame of reference is also inertial. (Note that this excludes the car in the example above, whose velocity is not constant, and the merry-go-round, which is rotating.) That's because any object that is observed to move at constant velocity with respect to the first frame of reference will also be observed to move at constant velocity — albeit at a *different* constant velocity — with respect to the second frame of reference. So if Newton's First Law holds with respect to the first frame, it will also hold with respect to the second.

In summary: Some frames of reference (namely, the inertial frames) are better than others, in the sense that the laws of physics take a much simpler form with respect to them than with respect to noninertial frames; in particular, Newton's First Law holds. But any *inertial* frame of reference is as good as any other, at least as far as Newton's First Law is concerned.

Galileo went much further: he guessed that any inertial frame of reference is as good as any other, not merely as far as Newton's First Law is concerned, but as far as *any* law of Nature is concerned. Here is how he put it in his *Dialogue Concerning the Two Chief World Systems* (1632):

> Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a narrow-mouthed vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in

> throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal ... When you have observed all these things carefully ... have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still.

Or, as we would put it in more modern (but much less picturesque) language:

> **The Principle of Relativity.** The laws of physics are the same with respect to all inertial frames of reference.

Note the key word "inertial": without it, the principle would simply be false, as our example of the accelerating car (or the rotating merry-go-round) shows.

What happens, for example, if a ball is dropped from the top of the mast of a ship? If the ship is at rest, obviously the ball will fall at the foot of the mast. But what if the ship is moving forwards? One's first guess might be that the ball will fall somewhere *behind* the foot of the mast. But this turns out not to be so: provided that the boat is moving at constant velocity (that is, "so long as the motion is uniform and not fluctuating this way and that"), the ball will again fall at the foot of the mast. Indeed, we can deduce this prediction from the Principle of Relativity. For if the earth frame of reference is inertial (which it is, to a good approximation) and the boat is moving at constant velocity (and nonrotating) with respect to the earth, then the boat frame of reference is also inertial. So we can apply, with respect to the boat frame of reference, all the laws of physics that we habitually apply with respect to the earth frame of reference. In particular, balls dropped from the top of the mast of a stationary boat should fall at the foot of the mast. But with respect to the *boat* frame of reference, the boat *is* stationary! So a ball dropped from the top of the mast should fall at the foot of the mast.