MATHEMATICS 1302 (Applied Mathematics 2)
YEAR 2008–2009, TERM 2

PROBLEM SET #7

This problem set is due at the *beginning* of lecture on Thursday 19 March (either morning or afternoon). Only problems 1–4 will be assessed. PLEASE NOTE THE NEW ARRANGEMENTS!!!

Topics: Motion constrained to a planar curve, including moving constraints (continued). Motion constrained to a surface of revolution. Open systems ("systems with changing mass"): accretion of matter, rocket motion.

Readings:

- Handout #7: Motion constrained to a moving curve.
- Dr. Baigent's 1302 notes on "Motion on a surface in 3D" (handout).
- Dr. Baigent's 1302 notes on "Variable mass systems" (handout).

1. Do Problem 5 of Problem Set #6. (If you already did this problem last week, fine; otherwise just do it now. It will be assessed as part of *this week's* problem set.)

2. A bead slides under the influence of gravity on the frictionless interior surface of the paraboloid of revolution $z = (x^2+y^2)/2a = r^2/2a$. Use cylindrical coordinates (r, φ, z).
   (a) Find the equations of motion for the bead coordinates (r, φ). [Eliminate z and all constraint forces.]
   (b) Find two conserved quantities.
   (c) Find a closed equation of motion for r alone. [One of the conserved quantities may appear as a parameter in your equation.]
   (d) Find the speed v_0 at which the bead will move in a horizontal circle of radius r_0.
   (e) Find the frequency of small radial oscillations around the circular motion found in part (d).

3. An open railway car of mass M rolls frictionlessly on a horizontal track, and is acted upon by a horizontal force F_0. At $t = 0$ the car has velocity v_0, and rain begins to fall vertically with respect to the ground. Rainwater enters the car at a constant rate α (mass/time) and leaks out through a small hole in the bottom of the car at a constant rate β, with $\alpha > \beta$.

(a) Find the equation of motion of the car.

(b) Solve this equation to find the velocity as a function of time. [*Hint:* Use the method of integrating factors for solving first-order linear differential equations with non-constant coefficients.]

4. A uniform heavy chain of length a is partly sitting on a table, partly hanging down over the edge. Initially a part of length b ($< a$) hangs over the edge (with zero initial velocity), while the remaining part of length $a - b$ is coiled up at the edge of the table.

   (a) Find the equation of motion for the amount $x(t)$ hanging over the edge.

   (b) Solve this equation to find the speed of the chain when the last link leaves the edge of the table. [*Hint:* $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}$.]

   (c) Is energy conserved? Explain physically why or why not.

   Same problem, but this time the part of the chain sitting on the table is stretched out to its full length in the direction perpendicular to the edge of the table. The table is frictionless.

   (d) Find the equation of motion for the amount $x(t)$ hanging over the edge.

   (e) Solve this equation to find the speed of the chain when the last link leaves the edge of the table.

   (f) Is energy conserved? Explain physically why or why not.

5. Find and solve the equation of motion for a raindrop falling through mist, collecting mass as it falls, under each of the following two hypotheses:

   (a) Assume that the raindrop remains spherical and that the rate of accretion of mass is proportional to the drop's cross-sectional area multiplied by its speed of fall. Show that if the drop starts from rest when it is infinitely small, then its acceleration is constant and is equal to $g/7$.

   (b) Assume that the raindrop remains spherical and that the rate of accretion of mass is proportional to the drop's surface area. Show that if the drop starts from rest when it is infinitely small, then its acceleration is constant and is equal to $g/4$.