MATHEMATICS 1302 (Applied Mathematics 2)
YEAR 2008–2009, TERM 2

PROBLEM SET #3

This problem set is due at the *beginning* of problem class on Wednesday 11 February.

Topics: Work done by a force. Work-energy theorem. Conservative and nonconservative forces; potential energy. Conservation of energy in two- and three-dimensional motion.

Readings:

- Feynman, vol. 1, Section 4–1 (handout).
- Kleppner and Kolenkow, Sections 4.1–4.12 (handout). The problems at the end of this chapter are excellent — I am assigning here a few of them, but I strongly urge you to work additional ones as practice.
- You may also wish to look at a textbook on vector calculus concerning the gradient, curl and Stokes' theorem. We will discuss this in a bit more detail next week; and then you will study it in even more detail in MATH 1402 (see the notes for Weeks 5–7 at `http://www.ucl.ac.uk/~ucahdrb/MATH1402/index.htm`).

1. Kleppner and Kolenkow, Problem 4.1. (*Hint:* First use conservation of energy to find the block's speed when it reaches point a. Then use $\mathbf{F} = m\mathbf{a}$ to find the normal force on the block. You will need to recall the kinematics of nonuniform circular motion: when a particle moves in a circle of radius R with speed $v(t)$, the radial component of its acceleration is v^2/R inwards, and the tangential component of its acceleration is dv/dt.)

2. Kleppner and Kolenkow, Problem 4.4. (*Hint:* Use conservation of energy and conservation of momentum. Note that the velocity v being requested in this problem is the cube's velocity *relative to the earth* as it leaves the block.)

3. Kleppner and Kolenkow, Problem 4.5. (*Hint:* Use conservation of angular momentum to find the mass' speed v when the string has length ℓ, assuming that it had speed v_1 when the string had length ℓ_1. Using this, find the tension in the string when the string has length ℓ. Integrate this from $\ell = \ell_1$ to $\ell = \ell_2$ to find the work done.)

4. A small block of mass m starts from rest and slides down the surface of a frictionless solid sphere of radius R that is bolted to the earth (Kleppner + Kolenkow, Problem 4.6). Measure angles θ from the vertical, and measure potential energy from the top. Find:

   (a) the particle's potential energy as a function of angle;

   (b) the particle's kinetic energy as a function of angle;

   (c) the particle's speed as a function of angle;

   (d) the radial and tangential components of the particle's acceleration, as a function of angle;

   (e) the angle at which the mass flies off the sphere.

   [*Hint for part (d):* You will need once again the kinematics of nonuniform circular motion, as discussed in the hint to Problem 1. The radial acceleration is easy to compute here, using your answer from part (c). There are at least two different approaches to finding the tangential acceleration: Probably the simplest is to apply $\mathbf{F} = m\mathbf{a}$, using tilted (radial-tangential) axes. Alternatively, use $a_{\tan} = dv/dt$ together with the chain rule

$$\frac{dv}{dt} \;=\; \frac{dv}{d\theta}\frac{d\theta}{dt} \;=\; \frac{dv}{d\theta}\frac{v}{r} \;=\; \frac{1}{r}\frac{d}{d\theta}\left(\frac{1}{2}v^2\right) \; .$$

   (But make sure you understand every step in this chain of equalities.) You might find it enjoyable to compute the tangential acceleration in *both* of these (very different) ways, and see if you get the same answer. *Hint for part (e):* First find the normal force as a function of θ, using $\mathbf{F} = m\mathbf{a}$ combined with your answers from part (d).]

5. An MI6 agent with some training in physics has been assigned to scrutinize the activities of an ultramodern trawler operating off the coast of Iran. Here is an excerpt from the report he filed:

   > Aboard the trawler is a long slide, kept covered with running water so that it is practically frictionless. The slide is 25 m long and is inclined at an angle $\theta = \arcsin \frac{4}{5}$ relative to the horizontal; the top of the slide is at the back of the trawler and the bottom of the slide is near the ship's midpoint. When a fish (mass 2 kg) is caught, it is brought to the top of the slide and released from rest, whereupon it slides down into the hold.
   >
   > Last Monday I took some films of this operation, from my vantage point in a dinghy at rest. The trawler was steaming forward at 18 m/sec. Analyzing the films frame by frame, I found that a fish starts down the slide with a speed of 18 m/s (relative to me) and reaches the bottom with a speed of 34 m/s (again relative to me). A quick calculation has convinced me that conservation of energy does not explain this gain in speed.
   >
   > In view of the military implications of this result, I recommend that a clandestine operation to capture this trawler be planned. Should this be unfeasible, I recommend that the trawler be neutralized by a cruise-missile strike.

(a) Repeat the agent's calculation: With respect to the agent's frame of reference, find the fish's kinetic energy and potential energy at the beginning and end of its motion. Show that the fish's total energy increases by 432 joules during the course of this motion. (Take $g = 10$ m/s^2.)

(b) Reanalyze the problem using the frame of reference of the trawler: find the fish's velocity vector, kinetic energy and potential energy at the beginning and end of its motion. Show that the fish's total energy is conserved.

(c) Find the force (both components) that the slide exerts on the fish. (Does it matter whether one uses the agent's frame of reference or the trawler's? Why or why not?)

(d) Find the time required for the fish to travel down the slide. (Does it matter whether one uses the agent's frame of reference or the trawler's? Why or why not?)

(e) Find the displacement vector of the fish with respect to the agent's frame of reference.

(f) Compute the work done on the fish by the slide, with respect to the agent's frame of reference. Show that this accounts for the "missing" energy in part (a).

6. Recall that the gravitational force between masses m_1 and m_2 located a distance r apart is $F = Gm_1m_2/r^2$ (attractive).

(a) How much work is required to lift a mass m from the earth's surface to a height h above the earth's surface? Let R_0 denote the radius of the earth, and do *not* make the approximation that $h \ll R_0$.

(b) How much work is required to lift a mass m from the earth's surface to infinitely far away from the earth?

(c) Suppose that the mass m is launched straight upwards with an initial velocity v_0. How big does v_0 have to be in order for the mass to reach a height h above the earth's surface before falling back down? Do *not* make the approximation that $h \ll R_0$.

(d) How big does v_0 have to be in order for the mass to keep travelling away from the earth forever? (This velocity is called *escape velocity*.)

(e) Now go back to part (a), and make the approximation $h \ll R_0$. Show how your formula for the work reduces to the standard expression mgh. [*Hint:* Put $1/R_0$ and $1/(R_0 + h)$ over a common denominator, then approximate this denominator. What is g in terms of G and the mass and radius of the earth?]