MATHEMATICS 1302 (Applied Mathematics 2)
YEAR 2008–2009, TERM 2

PROBLEM SET #1

This problem set is due at the *beginning* of problem class on Wednesday 28 January.

Topics: Vector algebra and kinematics (review). Newton's first and second laws (review). Galileo's principle of relativity. Newton's third law. How to set up mechanics problems.

Readings:

- Reread the chapter on vector algebra and kinematics in any of the recommended textbooks.
- Kleppner and Kolenkow, Sections 2.1, 2.2, 2.4, 2.5 and problems (handout). The problems at the end of this chapter are excellent — I am assigning here a few of them, but I strongly urge you to work additional ones as practice.
- Feynman, Sections 12–1 through 12–4, and 12–6 (handout). Note especially his cautionary comments regarding the empirical law for friction (Section 12–2).
- Handout #1: Newton's First Law and the Principle of Relativity.
- Handout #2: Newton's Second Law.
- Handout #3: Newton's Third Law.
- Handout #4: How to Do Mechanics Problems.

1. A particle moves in three-dimensional space according to

$$\begin{aligned} x(t) &= A\cos\omega t \\ y(t) &= B\sin\omega t \\ z(t) &= Ct \end{aligned}$$

where A, B, C and ω are constants. (Let's assume for simplicity that $A, B, C, \omega > 0$.)

(a) What are the dimensions of A, B, C and ω? (Recall that the dimension of any physical quantity will be of the form $L^a T^b M^c$, where L denotes length, T denotes time, M denotes mass, and a, b, c are some exponents.)

(b) Sketch this motion, and describe it in words.

(c) What are the physical/geometrical meanings of A, B, C and ω? Are your answers consistent with the dimensions you attributed to A, B, C and ω in part (a)?

(d) Compute the velocity $\mathbf{v}(t)$, and describe it in words.

(e) Compute the speed $v(t)$, and sketch it roughly as a function of t. (Note that there will be three slightly different situations according as $A > B$, $A = B$ or $A < B$.)

(f) Compute the acceleration $\mathbf{a}(t)$, and describe it in words.

2. A horse is urged to pull a wagon. The horse refuses to try, citing Newton's third law as his defense: "The pull of the horse on the wagon is equal in magnitude (and opposite in direction) to the pull of the wagon on the horse. If I can never exert a greater force on the wagon than it exerts on me, how can I ever start the wagon moving?"

   Obviously, the horse has misunderstood Newton's third law (although he has quoted it correctly). Your task is to explain to him exactly what Newton's third law *does* imply in the context of this problem, and to do so *clearly*, *precisely*, and in full English sentences. Start by drawing single-body diagrams for the wagon, the horse, and the earth. (If you're not sure what role the earth plays, ask yourself what happens if the horse is standing on an icy pond; and then ask in what way the situation is different if the horse is standing on an ordinary road.) What type of frictional force acts between the horse and the earth? Between the wagon and the earth? Explain the difference.

3. A plumb bob (i.e. a heavy metal ball attached to a cord) is hung from the ceiling of a car. The car is moving along a straight flat road.

   (a) If the car is at rest, at what angle θ relative to the vertical will the plumb bob hang? (This is easy.)

   (b) Suppose now that the car is moving forward at a constant velocity v. Find the angle θ (relative to the vertical) at which the plumb bob will hang — express it as a function of v. (Note that you can answer this *without* applying Newton's laws of motion. What principle should you use?)

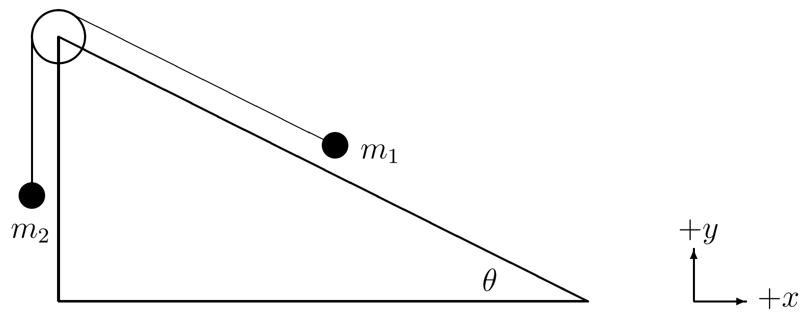
   (c) Now we will apply Newton's laws of motion. Draw a careful diagram showing the forces acting on the metal ball.

   (d) Suppose that the plumb bob is observed to hang at a constant (that is, unchanging) angle θ relative to the vertical. Show that the car must be moving with a constant acceleration a, and determine a. (*Warning:* Make sure that you are using an *inertial* frame of reference in performing this analysis.)

   (e) Suppose that, starting from rest, the car can reach a speed of 100 km/hr in 10 seconds by proceeding at an appropriate constant acceleration. At what angle θ will the plumb bob hang?

4. Kleppner and Kolenkow, Problem 2.9.

5. A block of mass m_1 lying on a smooth (frictionless) inclined plane of angle θ is connected by a cord over a massless frictionless pulley to a second block of mass m_2 hanging vertically. The inclined plane is bolted to the earth, so for all practical purposes it does not move.



(a) If block 2 has a vertical acceleration a (obviously the horizontal component of its acceleration is zero), what are the horizontal and vertical components of the acceleration of block 1? Be very careful about signs.

(b) Draw a single-body diagram indicating the forces on block 1. Then do the same for block 2.

(c) What is the acceleration of each body? What is the tension in the cord?

(d) Under what conditions does block 2 accelerate downwards? Upwards? Move at constant velocity? Check in particular the limiting cases $\theta = 0$ and $\theta = \pi/2$. Is your answer for the acceleration a physically reasonable in the way it depends on m_1, m_2 and θ?

6. Kleppner and Kolenkow, Problem 2.13. (*Hint:* Draw careful single-body diagrams for M_1, M_2 and the lower pulley. Then work out the geometrical constraint among these three objects.)

7. Kleppner and Kolenkow, Problem 2.16. (*Hint:* Draw a single-body diagram showing the forces acting on the block, and write the horizontal and vertical components of $\mathbf{F} = m\mathbf{a}$. Now work out the geometrical constraint relating a_x to a_y, and solve all the equations.)

8. Kleppner and Kolenkow, Problem 2.18. (*Hint:* Draw careful single-body diagrams for the painter and the platform. Don't forget the normal force acting between the platform and the painter.)