MATH1302, Question Sheet 6

Questions 1, 2 and 4 to be handed in Tuesday 4 March before the lecture.

Qu 1 A fisherman, returning from a fishing trip, turns off his engine and lets his dinghy drift slowly at speed U the last few metres to the river bank. When still a few metres from the bank he throws his rucksack mass m towards the bank (in the direction that the boat is drifting) at speed V relative to the boat and at an angle α to the horizontal. Given that the combined mass of the fisherman, the dinghy and its contents, including the rucksack, is $M+m$, find the condition that he will reach the bank without having to restart the engine. (Assume that the river is stationary.)

Qu 2 A runaway railfreight carriage filled with sand is travelling freely down a long, straight and constantly inclined railway track. The empty carriage has mass M, it initially holds S_0 mass of sand, and the inclination of the track to the horizontal is α. There is a crack in the carriage which allows sand to leak out at a constant rate. There is also a partially failed brake that provides a constant resistive force B to slow the carriage. Find the speed of the carriage from rest at time $t < T$ where T is the time taken for all the sand to leak out.

Qu 3 A particle is moving along the $x-$axis under a constant force F gains mass by collecting material that is moving along the $x-$axis with velocity u. If m and $v = dx/dt$ are the mass and the velocity of the particle at time t show that

$$\frac{d}{dt}(mv) = F + u\frac{dm}{dt}.$$

If $u = 0$ and $m = M + kx$ where $M, k > 0$ are constants, show that $kx = \sqrt{(M^2 + kFt^2)} - M$.

Qu 4 The total mass of a rocket is $M_0 + M_1$, including fuel of mass M_1. The fuel is burnt at a constant rate λ as is emitted as a stream of gas at a speed u relative to the rocket. The rocket is fired vertically upwards under uniform gravity g. If the mass of the fuel and the speed of the rocket at time t are $m(t)$ and $v(t)$ respectively, derive the equation

$$\begin{aligned}(M_0 + m)\frac{dv}{dt} + u\frac{dm}{dt} &= -(M_0 + m)g, \\ \frac{dm}{dt} &= -\lambda.\end{aligned}$$

Hence show that for $t < M_1/\lambda$

$$v = u \log\left(\frac{M_0 + M_1}{M_0 + M_1 - \lambda t}\right) - gt,$$

Qu 5 A raindrop falls vertically through a cloud of water particles which are at rest, and accumulates particles at a rate kv units of mass per unit time when its speed is v. If the raindrop starts from rest and with mass M, show that its speed v after falling through a distance x satisfies

$$v\frac{dv}{dx} + \frac{kv^2}{M + kx} = g.$$

Hence find v as a function of x.