MATH1302, Question Sheet 5

Questions 2, 5 and 6 to be handed in Tuesday 26 February before the lecture.

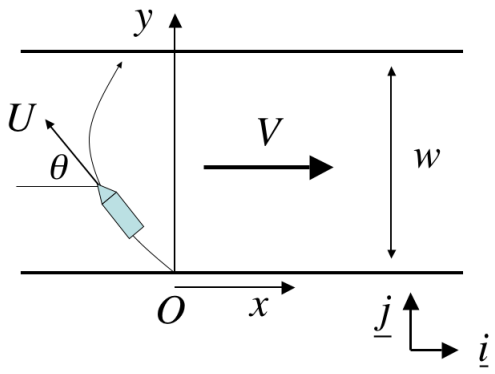
Qu 1 Two particles of masses m, m' move along the $x-$axis under their mutual attractive force F. The particles start at $x = 0$ with velocities u, u' respectively with $u' > u$. Show that their centre of mass moves with constant velocity and find that velocity. If the force F is constant, show that they will collide in finite time and find that time.

Qu 2 (Two body problem) Two stars A, B of masses m_A, m_B move under mutual gravitational attraction (Gm_Am_B/r^2 where r is their separation). Prove that their centre of mass moves with constant velocity and show that

$$\frac{d^2r_A}{dt^2} = -\frac{Gm_B^3}{(m_A+m_B)^2r_A^2}, \quad \frac{d^2r_B}{dt^2} = -\frac{Gm_A^3}{(m_A+m_B)^2r_B^2},$$

where r_A (r_B) is the distance of A (B) from the centre of mass.

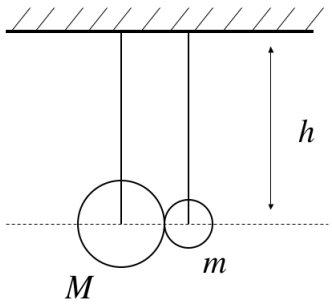
Qu 3 A river w metres wide flows at speed $V(y) = V_0y(w-y)$ to the right, where y is the perpendicular distance from the bank (as shown). The captain of a ferry can control the direction of the boat via the rudder, but its speed in still water is a fixed U. If the captain steers the boat to point at a constant angle θ relative to the bank, show that the velocity of the ferry relative to the ground is $(V(y) - U\cos(\theta))\underline{i} + U\sin(\theta)\underline{j}$. Where does the ferry arrive on the opposite bank relative to its starting point O?



Qu 4 A marble dropped vertically onto smooth ground rebounds vertically upwards with speed U. If $e \in (0,1)$ is the coefficient of restitution, show that the time taken from when it was dropped to the nth rebound is $\dfrac{U}{g}\dfrac{(1+e-2e^n)}{e(1-e)}$. How long does it take before the ball has effectively come to a rest?

Qu 5 Three snooker balls A, B, C each of mass m lie in order on a straight line. Balls B, C are initially at rest and A has speed U directly towards B. If the coefficient of restitution at each impact is $2/5$ show that there will be just three impacts and find the final speed of each ball.

Qu 6



Consider the version of Newton's cradle shown: two masses, m, M hang vertically so that their centre of masses are vertically below their supports and so that they are just touching. The mass m is pulled upwards (so that the string remains taut) to a height $h/2$ from the lowest point and released. If the coefficient of restitution between the balls is $e = m/M$ find the speed of the mass m after the first and second impacts.