MATH1302, Question Sheet 1

Questions 1,3, 4 to be handed in Tuesday 22 January before the lecture.

Qu 1 A particle of unit mass moves in 3 dimensional space so that its position vector at time t is $\underline{r}(t) = a\cos\omega t\underline{i} + b\sin\omega t\underline{j} + ct\underline{k}$ where $a, b, c > 0$ and $a > b$. Find (i) the particle's velocity, (ii) its acceleration, (iii) sketch its speed as a function of t and finally (iv) sketch also its path.

Qu 2 An airplane flying in a straight line at constant height H and speed U needs to drop supplies on a target at ground level. Given that the airplane eventually flies directly over the target, what distance before the target does it need to release the supply package if it is to hit the target? (Neglect air resistance.)

Qu 3 A ball mass m slides down a smooth (fixed) plane inclined at an angle α to the horizontal. The height of the plane from the lowest point is h. Taking unit vectors $\underline{e_1}$ parallel to the plane and $\underline{e_2}$ perpendicular to the plane, and $d(t)$ the distance of the ball down the slope at time t, write down (i) the acceleration $\underline{a}$ of the ball and (ii) the total force $\underline{F}$ acting on it. Hence show (iii) that $\ddot{d} = g\sin\alpha$. What is (iv) the magnitude of the reaction force on

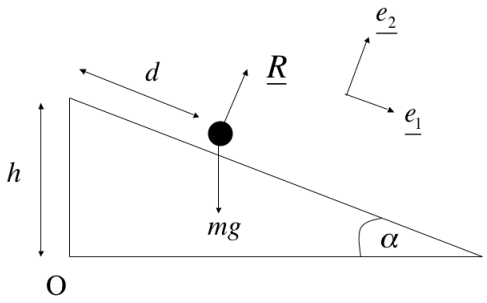

Figure 1: Figure for Qu 3

the ball from the plane? If the ball starts from the top of the slope at rest, find (v) the distance travelled down the slope in time t and also (vi) how long it takes for the ball to reach the bottom of the slope.

Qu 4 A heavy particle is fired with speed U from the base of an inclined plane. The plane is inclined at an angle α to the horizontal. Find the distance up the plane that the particle lands as a function of the angle θ between the inclined surface of the plane and the initial direction of the particle. (Neglect air resistance.)

Qu 5 A car of mass m breaks to stop by a combination of Coulomb's and Stokes' frictional damping, so that the total resistance is $-\mu mg - k\dot{x}$ ($\dot{x} > 0$), where x is the distance from

the point at which the brakes are applied, and $\mu > 0$, $k \geq 0$ constants. Show that the car breaking at v_0 will come to rest at a time

$$T = \frac{m}{k} \log \left(1 + \frac{k}{m} t_{\text{stop}} \right),$$

where t_{stop} is the stopping time for pure Coulomb damping ($k = 0$) from the same initial speed v_0.

Qu 6 A heavy particle is projected at speed U at an angle α to the horizontal. The particle is subject to air resistance which is experimentally found to vary proportionally to the square of the speed. Show that

$$\dot{\underline{v}} = -\frac{g}{V^2} |\underline{v}| \underline{v} - g\underline{j},$$

where V is the maximum speed of the particle. If $\alpha = \frac{\pi}{2}$ (so that the particle is projected directly upwards), find the maximum height reached and the time taken to reach it. What is the speed of the particle when it returns to the horizontal?