MATHEMATICS 0054 (Analytical Dynamics)
YEAR 2021–2022, TERM 2

HANDOUT #4: HOW TO DO MECHANICS PROBLEMS (OR, "THE GOSPEL ACCORDING TO SOKAL")

Each of the following steps involves drawing a picture. Draw each one as a **separate** picture.

STEP 1. Draw a general diagram of the system. You may need this picture later, e.g. for doing trigonometry, for determining geometrical constraints like "acceleration of particle 1 = acceleration of particle 2", etc.

STEP 2. The key step: Decide what are the individual "bodies" composing the system, and for each body **separately**, draw a single-body diagram showing all of the forces acting **on** that body. More precisely:

For each one of the forces acting **on** the given body, draw an arrow indicating the direction of the force. A force is a tangible "push or pull" exerted *by some other object* **on the given body**. For each force you draw, you **must be able to specify precisely**:

- what kind of a force it is (e.g. normal force, frictional force, gravitational force, tension)[1]; and
- what other object is exerting it (e.g. the table, the road, the earth, the string).

If you can't specify these two things, then the alleged "force" probably isn't one, and it doesn't belong on your diagram.

Some do's and don'ts (more precisely, some **DON'T**s):

- The acceleration is **NOT** a force; **DO NOT** put it on your single-body diagram. (The acceleration is the aspect of motion *caused by* the forces.) If you wish, you may put an arrow *off to the side* of your diagram, to indicate the direction of the acceleration (if you know it).
- The "net force" is **NOT** a distinct tangible force; **DO NOT** put it on your diagram. (The net force is simply the vector sum of all the individual forces that you *do* put on your diagram. To count it as an individual force would be like listing "net income" on your tax return alongside "wages", "interest" and other individual types of income — it's double-counting.)

[1] **Note:** "Centripetal force" is *not* a legitimate answer to this question. The word "centripetal" tells which direction the force acts in (namely, towards the center of some circle) but does not answer the question of what *kind* of a force it is. In fact, you will be best off if you avoid *ever* using the word "centripetal".

- **DO NOT** put on your diagram any forces exerted *by* the given body on some other body.
- **DO NOT** put on your diagram any forces which act on some *other* body. (Example: Block A lies on top of block B which lies on a table. The "weight of block A" is just a shorthand way of saying "the gravitational force exerted by the earth on block A". This force belongs, therefore, in block A's single-body diagram. It does *not* act on block B, so it does *not* belong in block B's single-body diagram.)
- **There is no such thing** as a "centrifugal force"; **DO NOT** put one in your single-body diagram.[2]
- The normal force is **NOT** always equal to the weight; indeed, there is *no* general rule for guessing the normal force at the beginning. Therefore, **DO NOT** try to specify the numerical value of a normal force at the beginning of the problem; in fact, do not make *any* assumptions about its magnitude. Just give it a *letter name*, and write the equations in terms of this letter name; you will solve for its value *at the end*. (Same comment for tension in a string and for static friction.)
- **DO NOT** try to resolve forces into components yet. (That will come later; to do it now just gets you confused.) Just draw the force in whatever direction it really acts. If you know what line the force acts along (e.g. vertical) but are not sure in which direction it acts (e.g. up or down), just choose one of the two directions and make the *convention* that a force in that direction will be called positive. Then, if after solving your equations you find that this force turns out negative, so be it: that just means that the force points in the direction opposite to the one that you chose to call positive.

STEP 3. After you have drawn your single-body diagram(s), do the following **separately** for **each** single-body diagram:

1) Choose x and y axes. You may choose whatever directions you find most convenient. **Make sure to label which directions you consider $+x$ and $+y$**; and stick to those choices once you have made them.
2) Resolve each force into its x and y components.
3) Compute $F_{x,total}$ = sum of x components of all the individual forces acting *on* the given body. Then do the same for $F_{y,total}$.
4) Set $F_{x,total} = ma_x$. (Keep all the forces on one side of the equation, and the acceleration on the other — **do not** mix them up.) Then do the same for $F_{y,total} = ma_y$.

STEP 4. Now you will proceed to solve the set of equations you have determined:

[2] In fact, you will be best off if you avoid *ever* using the word "centrifugal".

1) Reread the problem to see which quantities are given and which ones need to be determined. Count equations and unknowns.

   a) If you have an equal number of equations and unknowns, then your system of equations probably has a solution; skip to item (2).

   b) If you have more equations than unknowns, then your equations may be redundant (i.e. one of them may be deducible algebraically from the others); this is annoying but harmless. However, it is more likely that your system of equations has *no solution at all*; this probably means that you made a mistake somewhere earlier in the problem. So proceed to item (2), but proceed cautiously: if you find that your system of equations has no solution, then go back and try to find your error (which may be either physical or mathematical).

   c) If you have fewer equations than unknowns, then you need to find an additional equation — and it must be one *which is not just a mathematical consequence of the others*. That is, it must express some additional *physical* idea. What could this equation be? Here are some possibilities:

      ∗ Did you use all of the components of $\mathbf{F} = m\mathbf{a}$?

      ∗ Did you use the fact that the tension in a string is the same at all points in the string?

      ∗ Did you use the fact that a string is unstretchable? This imposes constraints on the accelerations of the bodies attached to the string.

2) Solve simultaneously your system of equations, using any legitimate algebraic method you like. (For example, you can add or subtract the equations so as to eliminate one of the variables; or you can solve one of the equations for one of the variables and then plug this into the others; etc.)

3) **Check your answer mathematically** by making sure that it satisfies *all* of your equations.

4) **Check your answer physically** — does your answer make sense? For example:

   – Is the answer *dimensionally correct*? That is, when you write the *units* in which each quantity is measured (which will be some combination of metres, seconds and kilograms), are the units the same on both sides of the equation?

   – Does each quantity vary as a function of the other quantities in a sensible way? (Example: A car rounds a curve of radius R on a horizontal road at speed v; what is the minimum coefficient of static friction required to keep the car from skidding? Suppose that your answer was $\mu_{s,min} = gR/v^2$. This is dimensionally correct, but it is physically absurd: it says that the faster you go, the less friction you need; and the tighter a curve, the less friction you need. This is clearly backwards; this answer *cannot* be right.)

   – Is the sign ($+$ or $-$) of each quantity reasonable?

- If numerical values are involved, is the magnitude of each quantity reasonable? (Example: In a practical mechanics problem, a velocity of 10^{-6} m/sec or 10^6 m/sec is probably wrong; likewise for a force of 10^{-6} N or 10^6 N.)
- In simple special cases (e.g. $v_0 = 0$, $\theta = 90°$, etc.), does the answer reduce to what you knew previously?

Some reminders:

- Different bodies may have different masses. Be sure to give different names to those masses (e.g. $m_1, m_2, \ldots$) — don't just blindly call them all m.
- Different bodies may have different accelerations. Be sure to give different names to those accelerations (e.g. $a_1, a_2, \ldots$) — don't just blindly call them all a. [Often you will be able to find some equation relating these different accelerations, e.g. if two bodies are connected by an unstretchable string. But you have to *reason out* what the correct relation is, using the picture you drew in Step 1.]

THE GOSPEL ACCORDING TO SOKAL, SUMMARIZED: Between now and the final exam, do a lot of practice problems, and apply these principles **faithfully** and **strictly** to **every** problem. If you do this, I can assure you that you will do well in this course — and learn a lot of mathematics and physics in the process.

Additional very useful information on problem-solving strategies in mechanics can be found in David Morin, *Introduction to Classical Mechanics* (Cambridge University Press, 2008), Chapter 1.