Signal propagation in oscillatory media enabled by noise-induced excitability

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ABSTRACT

A noise-induced signal propagation is reported in oscillatory media with FitzHugh-Nagumo dynamics which is based on a noise-induced phase transition to excitability. This transition occurs via a noise-induced suppression of self-excited oscillations, while the overall phase-space structure of the system is maintained. The noise-induced excitability enables the information transport in the originally oscillatory media. We demonstrate this new feature by the propagation of a wave front and the formation of a spiral in a two dimensional lattice. These spatio-temporal structures transport information and can be observed only in the presence of suitable amount of noise and not in the deterministic self-sustained oscillatory system. Thus we extend classes of nonlinear systems with signal transmission properties also to oscillatory systems, which demonstrate a noise-induced phase transition to excitability. Further on, the mechanism of noise-induced excitability provides the opportunity to control the information transport by noise via a triggering mechanism, i.e. the information channel is switched on in the presence of noise and switched off in its absence.

Keywords: phase transition, noise-induced, excitability, spiral, front, propagation, information transport, spatial temporal

1. INTRODUCTION

Signal transmission in excitable media is a key element of the information exchange in neuronal networks and many other natural and technical systems like laser, chemical and biological systems.[1, 2] All excitable media have in common a threshold of excitation and three states: a "rest" state, an "excited" (or "firing") state and a "recovery" (or "refractory") state. The rest state can be simply an equilibrium (or fixed) point or a small-amplitude subthreshold limit cycle. Small perturbations below the threshold of excitation result in small-amplitude linear response near the rest state. Sufficiently large perturbations beyond the threshold enable the system to leave the rest state, to go through firing and recovery state before it reenters the rest state again. This strongly nonlinear behavior results in a large difference in the response to small changes in the input and is accomplished by a large excursion through the phase space, the so called spike. During a spike an input impulse can not perturb the system significantly. The system is very refractory during a spike and does not response to further perturbations, which means that it takes a certain recovery time before another excitation can initiate a second spike. In spatially extended excitable media, perturbations can come from neighboring areas, and propagate unperturbed over long distances. Hence, excitable systems are specially useful for signal transmission.

Our everyday experience knows noise only as a nuisance which hinders or destroys the information transport and normally one attempts to avoid it as much as possible. In contrast to that, under certain conditions noise is also able to improve the signal transmission or to enhance the order of a system. The unexpected and constructive influence of noise or irregular fluctuations on excitable systems is attracting an increasing interest. A well-known example of the constructive role of noise is the *Stochastic Resonance* (SR) effect. SR can be found e.g. in bistable and excitable systems and describes an improvement of the synchronization of the system output with the input

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by an optimal amount of noise.[3,4] Beside many other natural systems which demonstrate SR in experiments, SR can be found also in such a complicated system as the human brain.[5] Noise-enhanced signal transport is an extension of the classical SR to a spatially coupled ensemble. Noise-induced propagation has been reported in bistable[6] and excitable systems.[7]

The above-mentioned effects of noise commonly require special system properties (e.g. excitability or bistability) and noise do not change these immanent system properties. However, increasing noise can also lead to a new state, which is qualitatively different from the old one. This phenomena is known as *noise-induced transition* (NIT) in nonequilibrium systems and can be considered as a generalization of phase transitions in thermodynamic equilibrium systems. NIT are already known in zero dimensional systems with multiplicative noise, which leads to the appearance of additional extrema[8] (or the disappearance of existing ones[9]) in the probability distribution, to noise-induced oscillations,[10] or to noise-induced bistability in spatially extended systems.[11,12] In the latter, recent investigations have also shown *bona-fide transitions* caused by multiplicative noise from an excitable to an oscillatory regime via a renormalization of the parameters defining the local dynamics of the system.[13,14] This mechanism has also been found responsible of inducing excitability in bistable[15,16] and subexcitable[13] media.

In all those cases, however, noise has the expected role of inducing dynamical instability. In contrast to that we consider a system with noise induced stability. In this work, multiplicative noise leads to a phase transition from a self-sustained oscillatory to a noise-induced excitable state[17] and only due to this noise-induced system property, signal transport and spatial temporal pattern formation can be observed in an originally self-sustained oscillatory system, i.e. the excitable character of the noise-induced regime is responsible for the information transport through the system. This communication is structured as follows: after introducing the model equations in Section 2, we sketch the underlying phase transition in Section 3, and finally demonstrate in Section 4 the possibility of pulse propagation and spiral formation in a two-dimensional lattice.

2. THE MODEL

To demonstrate the phenomenon of noise-induced excitability (NIE) we consider a system of N locally coupled FitzHugh-Nagumo (FHN) elements in an oscillating state and under the action of multiplicative noise:

$$\dot{u}_i = \frac{1}{\varepsilon}(F(u_i) - v_i) + D_u\left(\bar{u}_i - u_i\right) \quad (1)$$

$$\dot{v}_i = cu_i + d + v_i\xi_i + D_v\left(\bar{v}_i - v_i\right). \quad (2)$$

The reaction term in the u - equation is given by:

$$F(u) = \begin{cases} -1 - u + b & u \le -\frac{1}{2} \\ gu + b + \frac{1}{2}(g-1) & -\frac{1}{2} < u < \frac{1}{g} - \frac{1}{2} \\ +1 - au + b - \frac{1}{2} + a(\frac{1}{g} - \frac{1}{2}) & u \ge \frac{1}{g} - \frac{1}{2}. \end{cases}$$

The coupling term in Eqs. (1)–(2) is given by $\bar{x}_i \equiv \frac{1}{\mathcal{N}}\sum_{j\in n.n} x_j$, where the sum runs only over the $\mathcal{N}$ nearest neighbors of site i, and $x_i = u_i, v_i$. In a neural context, $u(t)$ represents the membrane potential of the neuron and $v(t)$ is related to the time-dependent conductance of the potassium channels in the membrane.[1] The dynamics of the activator variable u is much faster than that of the inhibitor v, as indicated by the small time-scale-ratio parameter ε. Random fluctuations are represented by the $\delta-$correlated Gaussian noise $\xi_i(t)$, with zero mean and the correlation $\langle\xi_i(t)\xi_j(t')\rangle = \sigma_m^2\delta(t-t')\delta_{i,j}$. This multiplicative noise term is interpreted in the Stratonovich sense.[18] In what follows we consider a 2-dimensional lattice with fixed or periodic boundary conditions.

3. PHASE TRANSITION TO EXCITABILITY

The mechanism of a noise-induced phase transition in this model can be explained theoretically in the framework of a small-noise expansion,[18] which extracts the systematic contribution of the multiplicative noise that accounts for the excitability restoration. In particular, the multiplicative noise term $v_i\xi_i$ in Eq. (2) has a non-zero mean given by $\langle v_i\xi_i\rangle = (\sigma_m^2/2)v_i$ according to the small noise approximation.[18] Therefore, in the presence of fluctuations the effective local dynamics of the inhibitor variable is given by $\dot{v}_i = cu_i + d + (\sigma_m^2/2)v_i$, at first order in the noise intensity. The corresponding nullclines (curves with $\dot{u}_i = 0$, $\dot{v}_i = 0$) of an isolated oscillator for increasing multiplicative noise intensity are represented in the phase plane of Fig. 1. Without noise the nullcline for the slow variable v (curve 1) crosses the nullcline of the fast variable (inverted-N piecewise line) in its middle segment, so that the crossing point is an unstable steady state and the system exhibits an oscillatory behavior. An increase of the multiplicative noise intensity σ_m^2 leads to a tilting and shifting of the $v-$nullcline [curves 2-4 in Fig. 1]. As a result, for large enough σ_m^2 (in the present case for $\sigma_m^2 > 0.033$) the crossing occurs in the left segment of the u-nullcline and the fixed point becomes stable. One can conclude then that the NIE is realized by stabilizing a deterministically unstable fixed point of the dynamics, while preserving the overall phase-space structure that leads to large-amplitude pulses (but which will be then triggered only by above-threshold perturbations). In contrast to previous results on noise-induced excitability, spatial coupling is absolutely essential in this case, in order to prevent noise-driven oscillations from exciting the system and converting it back in an oscillator. In that sense, coupling plays here a role similar to that of standard phase transitions, suppressing fluctuations and coupling the stable regions. It prevents the system from visiting the whole available phase space and locks it close to the stable steady state (until a perturbation triggers an excitable spike). Noise-induced phase transitions between homogeneous phases have long been known to use the joint action of coupling and noise in this way[12, 18]; here we extend this fundamental mechanism to the field of excitable dynamics.

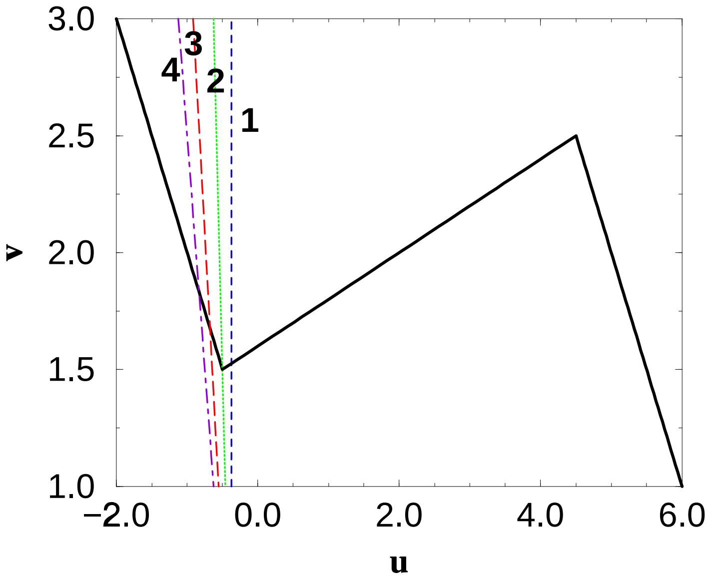

Figure 1. The nullclines of a single FHN oscillator. The inverted-N piecewise line corresponds to the noise independent nullcline of the activator u. The lines (1 - 4) describe the tilting of the inhibitor nullcline by increasing the noise intensity: $\sigma_m^2 = 0.0$ (1), $\sigma_m^2 = 0.0334$ (2), $\sigma_m^2 = 0.072$ (3) and $\sigma_m^2 = 0.1$ (4). Other parameters are $a = 1.0$, $b = 2.0$, $c = 0.2$, $d = 0.075$ and $g = 0.2$.

The noise-induced phase transition to excitability via the formation of clusters of stable elements can be seen in Fig. 2. For the numerical integration here and below we have used Heun's algorithm.[18] These figures depict the transient state from a random initial condition to the excitable state. A random realization of the uniform distribution with $u_i \in [-2.0; 6.0]$ and $v_i \in [1.0; 3.0]$ is chosen as random initial condition to demonstrate the strong attraction of the noise-induced fix point. In that way, 200×200 FHN oscillators with periodic boundary conditions occupy initially the whole phase space, as shown in Fig. 1. The multiplicative noise with $\sigma_m^2 = 0.072$ acts on every locally coupled oscillator and no further influences are present here. In the first figures a formation and growing of local excited clusters can be observed (time t up to 0.2 time units) until the suppression of the spikes due to the coupling and the multiplicative noise dominates. In this situation from $t = 0.6$ time units no spiking oscillator can be observed and every oscillator is attracted by the noise-induced fix point. As repeated in the beginning, such a behavior is typically for excitable systems and differs strongly from the originally deterministic self sustained oscillatory behavior of the system in the absence of noise ($\sigma_m^2 = 0.0$). The conditions for the noise-induced phase transition to excitability, especially the influence of the coupling, are discussed in detail in.[17]

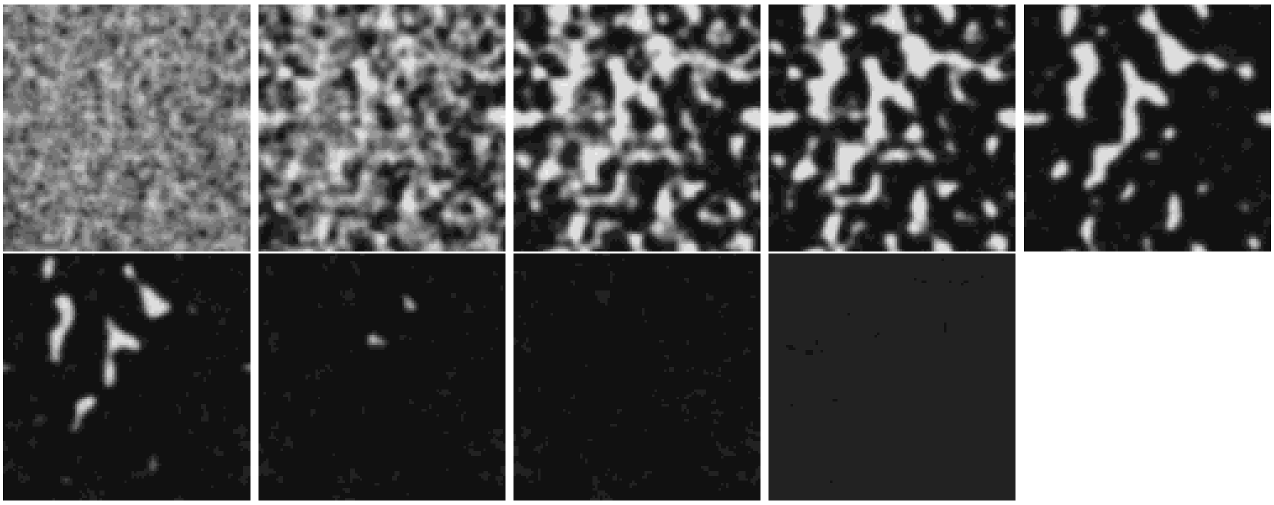

Figure 2. Snapshots of the numerically simulated activatory variable u of an array of 200×200 FHN's during the unperturbed transition from a random initial condition at $t = 0.0$ to the NIE state. The time increases from left to right and from top to bottom. First row: $t = 0.05$, 0.1, 0.15, 0.2 and 0.3, and second row: $t = 0.4$, 0.5, 0.6, and 6.0. Other parameters are $\varepsilon = 0.01$, $a = 1.0$, $b = 2.0$, $c = 0.2$, $d = 0.075$, $g = 0.2$, $D_u = 416$, $D_v = 64$, and $\sigma_m^2 = 0.072$.

4. SIGNAL PROPAGATION IN A TWO-DIMENSIONAL LATTICE OF LOCALLY COUPLED NOISE-INDUCED EXCITABLE FHN ELEMENTS

One of the main characteristics of excitable media is their ability to sustain lossless propagation of structures. This is, for instance, the way in which electrical pulses propagate through neural tissue in physiological systems.[1] The NIE regime reported here offers the possibility of a signal propagation through *oscillatory* media. Additionally, NIE allows the activation/deactivation of the excitable property, in such a way that information transport can be controlled by multiplicative noise. On the other hand, NIE opens the alternative to suppress and admit oscillations e.g. in pacemaker.

In the NIE region (i.e. for large enough σ_m^2), independently of the initial conditions every oscillator of the coupled ensemble moves to the NIE fixed point and remains there. As a consequence, the media can transmit an information signal. In this situation, the spatial temporal response of the system to a plane-wave perturbation is depicted in Fig. 3 (top row). The simulations start with uniform initial conditions $u_i = -0.7$ and $v_i = 1.7$ near the noise-induced fix-point at time $t = 0.0$ and include periodic boundary conditions. After a transient state (as in Fig. 2), at time $t = 10.0$ six rows of oscillators in the bottom are excited by setting these oscillators above the excitation threshold to $u_i = -0.3$ and $v_i = 1.3$, caused for instance by an external stimuli. Additionally the next 6 rows below the first one are set in a refractory state at $u_i = -1.0$ and $v_i = 2.0$ to suppress a bidirectional wave front propagation in the lattice with periodic boundary conditions. All other oscillators outside the noted excitation area are not excited by this external signal. These stimulations persist only 0.7 time units and after this stimulation time all oscillators remain unperturbed. We observe a clear propagation of the plane wave from the bottom to the top, i.e. an absolutely reliable signal transport across space takes place. The wavefront dies when it reenters the lattice on the bottom ($t = 28$) because the refractory time of the oscillators is greater then the traveling time of the wavefront across the lattice (600×600 oscillators in size in this case). The front hits oscillators in the refractory state of the noise-induced excitable dynamics and disappears, a typical feature of excitable systems, as described in the introduction. In the present case, the lattice size is too small to observe a reproducing wavefront with periodic boundaries. For larger lattices with periodic boundary conditions a permanent traveling wave appears.

As shown in Fig. 3, the sensitivity and the feature of information transport can be observed only in the noise-induced excitable regime (top row) and not in the deterministic self-sustained oscillatory system (bottom row). Both rows in Fig. 3 differ only by the multiplicative noise intensity (top: $\sigma_m^2 = 0.072$, hence the NIE regime; and bottom: $\sigma_m^2 = 0.0$, the self-sustained oscillatory regime). All other conditions like stimulation, coupling and local parameters are the same. In the self-sustained oscillatory regime [Fig. 3 (bottom row)] the small stimulation is not large enough to initiate stable propagating wavefronts. Only a phase-shift is caused, but no propagation can be observed. The coupling leads to a reduction of the phase-shift and hence to a lost of the information. The self-sustained oscillations appear at times $t = 18.8$ and $t = 31.0$, but they are not related to the information seeded by the initiated wavefront and only destroy the information transport.

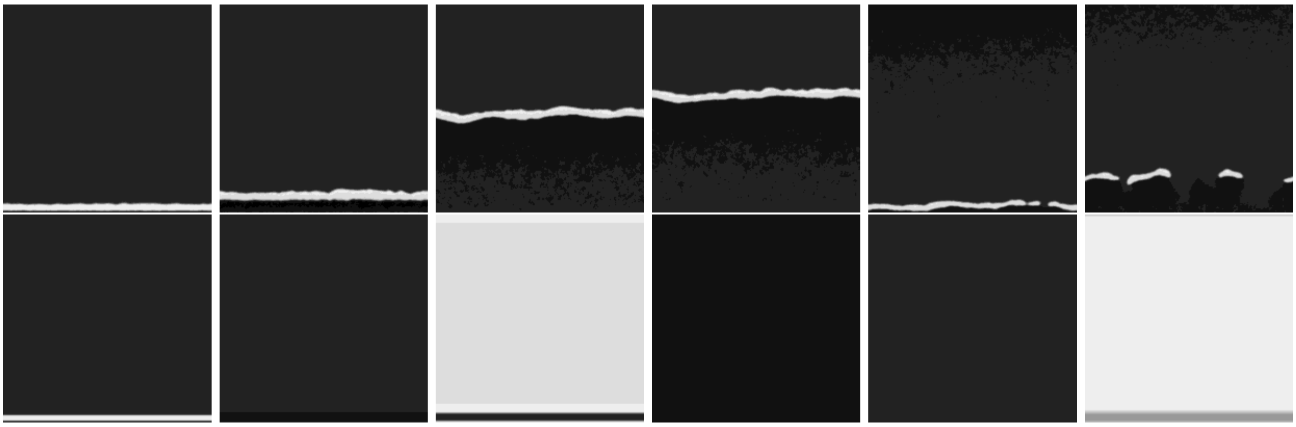

Figure 3. Snapshots of the numerically simulated activatory variable u for increasing time (from left to right). The top row shows the propagation of a wave front in an 600×600 array at time steps 11.0, 12.0, 18.5, 20.0, 28.0, and 31.0 time units in the noise-induced excitable regime ($\sigma_m^2 = 0.072$). The bottom row depicts the deterministic self-sustained oscillating regime ($\sigma_m^2 = 0.0$) at the same time steps with the same stimulation and demonstrates the necessity of the excitable property for the information transport. Other common parameters are $\varepsilon = 0.01$, $a = 1.0$, $b = 2.0$, $c = 0.2$, $d = 0.075$, $g = 0.2$, $D_u = 416$, and $D_v = 64$.

Spiral wave propagation can be also demonstrated in this system [Fig. 4 (top)]. The initialization of the spiral was implemented in a similar way to the above-mentioned wavefront, with the difference that the stimulation takes place in the middle of the lattice and only half of the rows are excited as one can see in the first plot on the top left of Fig. 4. In contrast to the wave propagation, in this example the lattice size (2600×2600 oscillators) is large enough for self-preserving spirals. One can clearly detect the minimal spatial distance between two consecutive spike chains, which is determined by the refractory time (caused by the nullclines of the FHN) and the propagation velocity (affected by the coupling strength). In the absence of multiplicative noise, on the other hand, the system exhibits a synchronous self-sustained oscillatory behavior and no wave propagation can be observed [Fig. 4 (bottom)]. The self-sustained oscillations take place at about time $t \approx 18$, $t \approx 31$, and $t \approx 44$. This means that the presence of multiplicative noise is crucial for information transmission in this system.

5. CONCLUSION

In summary, we have studied the information transport in noise-induced excitable media. The information transport is enabled by a noise-induced phase transition from a self-sustained oscillatory state to an excitable state. The appearance of noise-induced excitability is a collective effect, and occurs via a phase transition due to the joint action of coupling and multiplicative noise.[17] In contrast to standard phase transitions and other studies on excitable systems,[7,13–16] the increase of noise here enhances the stability in the system and restores excitable properties. This noise-supported excitability displays characteristic properties of standard spatially extended excitable media, such as wave propagation and spiral formation. With help of the noise-induced excitability, we could extend the system classes with a flexible and reliable information-transport ability to self-sustained oscillatory systems with an unstable fixed point. The interplay between excitable and oscillatory

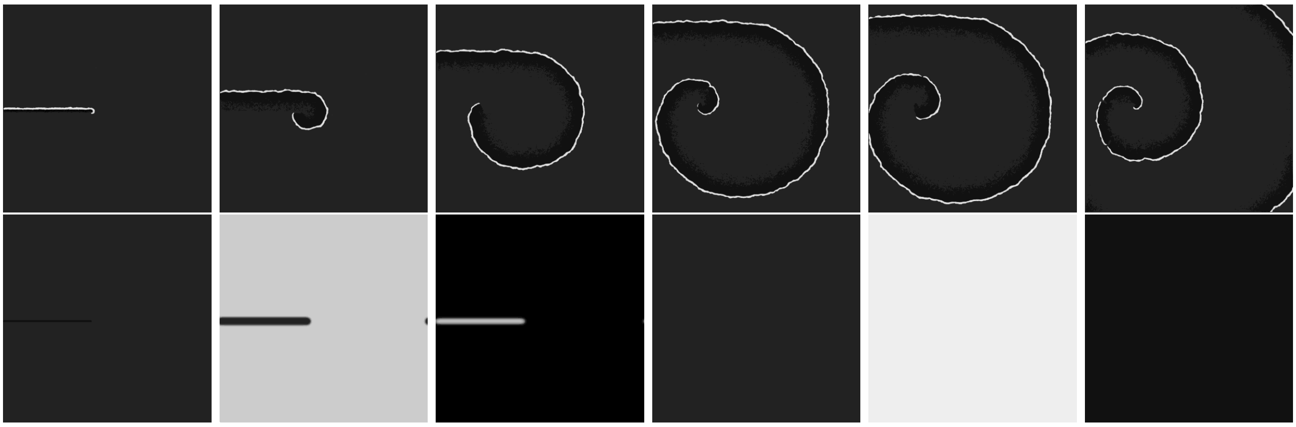

Figure 4. Snapshots of the numerically simulated activatory variable u for increasing time (from left to right). The top row shows the propagation of a spiral in an 2600×2600 array at time steps 12, 18, 32, 42, 44, and 60 time units in the noise-induced excitable regime ($\sigma_m^2 = 0.072$). The bottom row depicts the deterministic self-sustained oscillating regime ($\sigma_m^2 = 0.0$) at the same time steps and demonstrates the necessity of the excitable property for the spiral formation. All the other conditions and parameters are the same in both columns and in Fig. 3.

dynamics in noisy systems is a current important issue.[19] In particular, these theoretical findings suggest a possible mechanism to suppress undesirable global oscillations in neural networks (which are usually characteristic of abnormal medical conditions such as Parkinson disease or epilepsy), using the action of noise to restore excitability, which is the normal state of neuronal ensembles. In this way, noise could help to reconstruct the sensitivity of neuron ensembles for signaling. Additionally, the noise-induced excitability offers the opportunity to enable or suppress the information transport in a coupled system in a surprising way: an increase of noise enables the information exchange, whereas a lack of noise hinders it. Furthermore, this noise-induced excitability could open an opportunity to control pacemaker dynamics, so that the pacemaker fires periodically only in the absence of noise and falls silent in a noisy surrounding.

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