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Fluctuational transport of a Brownian particle in ratchet-like gravitational potential field

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Abstract

We treat analytically the problem of fluctuational transport of a Brownian particle in ratchet-like gravitational field in the presence of white noise. Given analytics allow to calculate the mean particle velocity depending on the particle mass and the noise intensity. We show that noise-induced current undergoes the reversal both with the increase of the particle mass and with the increase of the noise intensity. © 2001 Elsevier Science Ltd. All rights reserved.

In recent years, phenomena of noise-induced transport for Brownian particles has attracted considerable interest of many scientists, for the most part in the context of different biological and chemical problems (see, for example, [1–9]). Physical experiments demonstrating the possibility of such transport in a ratchet-like potential field created by laser beam are described in [10–12]. Some results of experimental investigations of collective transport in asymmetric periodic structures are presented in [13].

Systems in which noise-induced transport occurs are often called stochastic ratchets by analogy with mechanical device 'ratchet-and-pawl' described and considered by Feunman et al. [14]. In most works devoted to stochastic ratchets one-dimensional motion of a light Brownian particle in a saw-tooth potential field is considered. It is assumed that the medium, in which the particle moves, is sufficiently viscous and the potential is independent of particle mass. Under these assumptions the particle mass can be ignored. Taking account of particle mass [15–17,20] shows that in this case flux reversal is possible. At the same time a very important and interesting problem of motion of a Brownian particle along a hilly vibrated surface with periodic structure having regard to the force of gravity up till now has not been considered. This letter completes this omission.

So, we consider the case, when a Brownian particle moves along periodically vibrated hills having slopes of different steepness. The equation of motion of the particle can be written as

$$\mu\ddot{x} + \dot{x} + \mu f(x) = B\cos\omega t + \xi(t), \tag{1}$$

where $\mu = m/\gamma$, m the particle mass, γ the viscous friction factor, $mf(x)$ the force of gravity, $B\cos\omega t$ the force of inertia due to vibration, and $\xi(t)$ is the thermal noise. For definiteness, we set the function $f(x)$ to be proportional to the first two terms of the Fourier series for $f(x)$ corresponding to a saw-tooth potential, namely,

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$$f(x) = \sum_{n=1}^{2} \frac{1.1}{n\pi}\left[(a_1 + a_2)\sin\frac{2\pi n(x + x_0)}{L} - a_1 \sin\frac{2\pi n(x + x_0 - x_1)}{L} - a_2 \sin\frac{2\pi n(x + x_0 + x_2)}{L}\right], \tag{2}$$

where x_0 is chosen so that $f(0) = 0$. Plots of the functions $U(x) = \int f(x)\,dx$, $f(x)$ and $f'(x)$ for $x_1 = 0.8$, $x_2 = 0.2$, and $x_0 \approx 0.073$ are shown in Fig. 1.

The two-dimensional Focker–Planck equation corresponding to Eq. (1) is conveniently written as

$$\mu\frac{\partial w(x,y,t)}{\partial t} = -\sqrt{\mu}\left(y\frac{\partial w}{\partial x} - (\mu f(x) - B\cos\omega t)\frac{\partial w}{\partial y}\right) + \frac{\partial(yw)}{\partial y} + \frac{K}{2}\frac{\partial^2 w}{\partial y^2}, \tag{3}$$

where $y = \sqrt{\mu}\dot{x}$ and K is the noise intensity. If μ is sufficiently small, namely,

$$\mu^2 \max f'(x) \ll 1, \tag{4}$$

we can approximately reduce Eq. (3) to an one-dimensional Fokker–Planck equation for the probability density of the variable x, using the expansion with respect to the particle mass, much as this was first done by Stratonovich [18,19] and thereafter by us [20,21]. For moderately low frequencies ω this equation can be solved in the quasi-stationary approximation. Furthermore, if the vibration amplitude B is small, we can expand the solution as a series in B, retaining terms up to B^2, inclusive. As a result, we find the following series for the mean particle velocity:

$$\overline{\langle\dot{x}\rangle} \approx \frac{B^2 L}{2}\mu\Big[\tilde{G}_2 + \mu G_1 m_{11} + \mu^2(G_1 m_{21} + \tilde{G}_2 m_{10}) + \mu^3(G_1 m_{31} + \tilde{G}_2 m_{20}) + \mu^5(G_1 m_{41} + G_2 m_{30}) + \cdots\Big], \tag{5}$$

where

$$G_1 = \frac{L}{I_{10}I_{20}}, \quad \tilde{G}_2 = \frac{G_1}{\mu}\left[\frac{I_{11}}{I_{10}} - \frac{I_{21}}{I_{20}} - \frac{L}{K}\left(1 - \frac{2I_{30}}{I_{10}I_{20}}\right)\right], \quad m_{10} = \frac{I_{40}}{\mu I_{10}},$$

$$m_{11} = -m_{10}\left[\frac{I_{41}}{I_{40}} - \frac{I_{11}}{I_{10}} + \frac{2L}{KI_{20}}\left(\frac{I_{50}}{I_{40}} - \frac{I_{30}}{I_{10}}\right)\right], \quad m_{20} = \frac{I_8}{I_{10}},$$

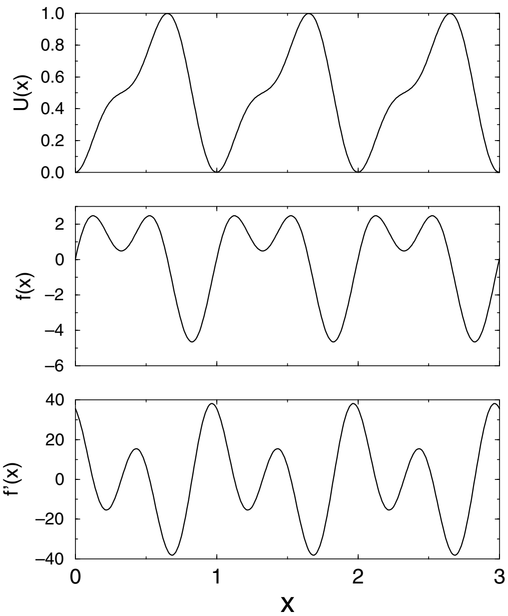

Fig. 1. Plots of the functions $U(x)$, $f(x)$, and $f'(x)$ for $x_1 = 0.8$, $x_2 = 0.2$, and $x_0 \approx 0.073$.

$$m_{21} = -\frac{I_{70}+I_9}{I_{10}} + \frac{I_{11}I_8}{I_{10}^2} - \frac{2L}{KI_{10}I_{20}}\left(I_{14} - \frac{I_{30}I_8}{I_{10}}\right), \quad m_{30} = m_{10}^2 + \frac{I_{12}}{I_{10}} + \frac{I_{16}}{I_{10}},$$

$$m_{31} = 2m_{10}m_{11} - \frac{I_{13}}{I_{10}} + \frac{I_{11}I_{12}}{I_{10}^2} - \frac{2L}{KI_{10}I_{20}}\left(I_{15} - \frac{I_{30}I_{12}}{I_{10}}\right) - \frac{I_{18}+I_{22}}{I_{10}} + \frac{I_{11}I_{16}}{I_{10}^2} - \frac{2L}{KI_{10}I_{20}}\left(I_{24} - \frac{I_{30}I_{16}}{I_{10}}\right),$$

$$m_{41} = 2\left(m_{10}m_{21} + \frac{I_8 m_{11}}{I_{10}}\right) - \frac{I_{19}+I_{23}}{I_{10}} + \frac{I_{11}I_{17}}{I_{10}^2} - \frac{2L}{KI_{10}I_{20}}\left(I_{25} - \frac{I_{30}I_{17}}{I_{10}}\right) - \frac{I_{28}+I_{29}}{I_{10}} + \frac{I_{11}I_{26}}{I_{10}^2} - \frac{I_{21}I_{26}}{I_{10}I_{20}} - \frac{2L}{KI_{10}I_{20}}\left(I_{31} - \frac{I_{30}I_{26}}{I_{10}}\right).$$

Expressions for integrals $I_{10}, I_{11}, I_{20}, I_{21}, I_{30}, I_{40}, I_{41}, I_{50}, I_{70}$ can be found in [21, pp. 31–35] (there $U(x)$ should be substituted by $\mu U(x)$).

Other integrals are the following:

$$I_8 = F[(3K/4)f'''(x)], \quad I_9 = F[(3/2)xf'''(x)], \quad I_{12} = F[f_1(x)], \quad I_{13} = F[(2/K)xf_1(x)],$$
$$I_{14} = FF[(3K/4)f'''(x)], \quad I_{15} = FF[f_1(x')], \quad I_{16} = F[(29K^2/48)f^{\mathrm{V}}(x)], \quad I_{17} = F[(K/8)f_2(x)],$$
$$I_{18} = F[(29K/24)xf^{\mathrm{V}}(x)], \quad I_{19} = F[(1/4)xf_2(x)], \quad I_{22} = F[(23K/8)f^{\mathrm{IV}}(x)],$$
$$I_{23} = F[7f(x)f'''(x) + 12f'(x)f''(x)], \quad I_{24} = FF[(29K^2/48)f^{\mathrm{V}}(x')], \quad I_{25} = FF[(K/8)f_2(x')],$$
$$I_{26} = F[(143K^3/288)f^{\mathrm{VII}}(x)],$$
$$I_{27} = F[(K^2/144)(485f(x)f^{\mathrm{VI}}(x) + 1677f^{\mathrm{V}}(x)f'(x) + 3081f^{\mathrm{IV}}(x)f''(x) + 1820(f'''(x))^2)],$$
$$I_{28} = F[(143K^2/144)xf^{\mathrm{VII}}(x)], \quad I_{29} = F[(485K^2/144)f^{\mathrm{VI}}(x)], \quad I_{31} = FF[(143K^3/288)f^{\mathrm{VII}}(x')],$$

where we have used following notations:

$$F[f(x)] = \int_0^L f(x)\,\exp\left(\frac{2(\mu U(x))}{K}\right)\mathrm{d}x,$$

$$FF[f(x')] = \int_0^L \int_0^x f(x')\,\exp\left(\frac{2(\mu U(x') - U(x))}{K}\right)\mathrm{d}x'\,\mathrm{d}x,$$

$$f_1(x) = 2f(x)f''(x) + (f'(x))^2,$$

$$f_2(x) = 23f(x)f^{\mathrm{IV}}(x) + 52f'(x)f'''(x) + 31(f''(x))^2.$$

The calculations performed allow us to find the dependencies of $v \equiv \gamma\overline{\langle\dot{x}\rangle}/B^2$ on K and μ. The dependencies of v on K for different values of the particle mass μ and for the different approximations with respect to μ are shown in Fig. 2. To calculate the value of current with different precision we have taken into account different number of terms in series (5). This figure clearly shows that increase of the noise intensity leads to the current reversal. This fact is important for the problem of control of particle motion. Interesting that in some cases (see Fig. 2(b), curve 4) the reversal of the fluctuational current occurs twice: for moderate values of the noise intensity and for large values. Our approximation is valid for large noise intensities only if the mass is small enough: the larger is the noise intensity, the smaller should be the mass to provide true result. Hence, the question whether indeed double reversal can be observed, calls for the further investigation.

Note that reversal can be obtained with increase both of the noise intensity and of the particle mass. To analyze the influence of the particle mass on the fluctuational transport in detail, we have calculated the dependence of v on the mass μ, also for different approximations with respect to μ.

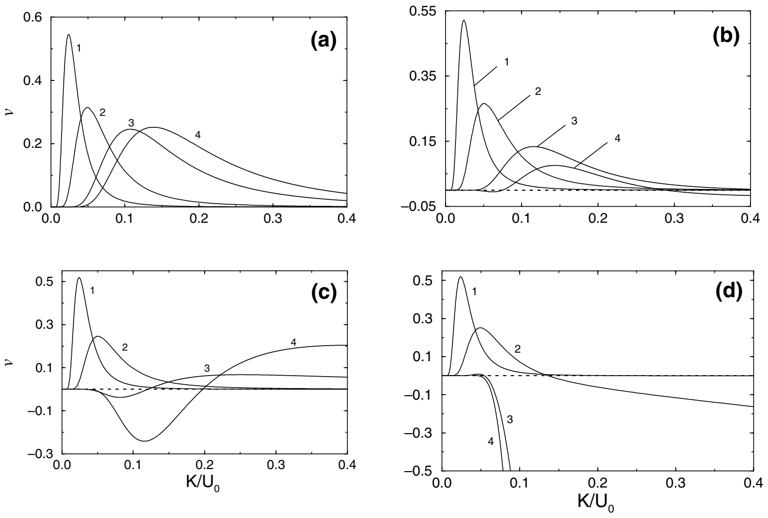

Fig. 2. Noise-induced current vs the intensity of the noise. The dependencies of v on K/U_0 for different number of terms in the expansion (5): (a) two terms; (b) three terms; (c) four terms; (d) five terms. The value of the μ is the following: 0.05 (curve 1), 0.1 (curve 2), 0.2 (curve 3), and 0.25 (curve 4).

These dependencies are given in Fig. 3, for values of $K = 0.05$, 0.1, 0.15 and 0.2. Analysis of this figure demonstrates that even for small values of the noise intensity we obtain the current reversal already in the third and higher approximations with respect to the mass. Note that for these noise intensities and for negative values of the current, the dependence of the current on the number of the approximation is not monotonic: fifth approximation is located above fourth one, but below third one. For large noise intensities $(K > 0.1)$ the situation changes: the larger is the number of approximation, the larger is the absolute value of the negative current.

In conclusion, we have shown that the flux reversal can occur with increasing the particle mass in a gravitational spatially periodic field as well as in a non-gravitational one [21,22]. Since such a gravitational field can be easily created, this result can be important for the possible experimental realization of the particle separation.

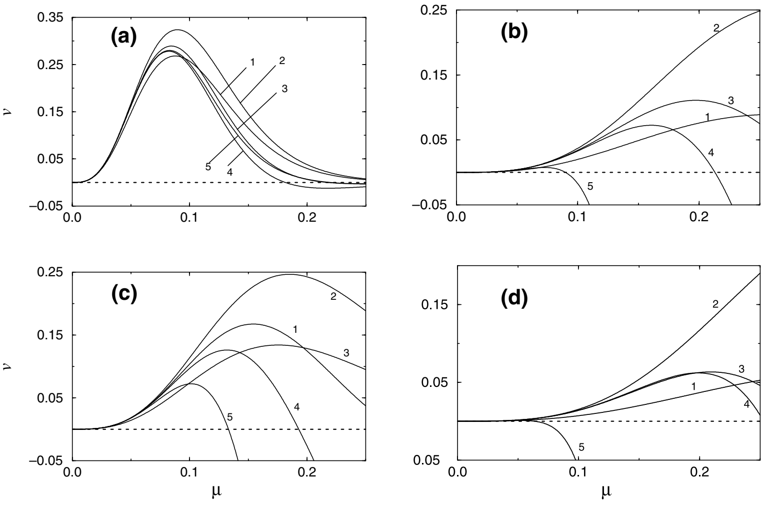

Fig. 3. Influence of particle mass on the noise-induced transport for different values of K/U_0: (a) 0.05, (b) 0.1, (c) 0.15, and (d) 0.2. The dependencies of v on μ are for different approximations (see series (5)): one term (curve 1), two terms (curve 2), three terms (curve 3), four terms (curve 4), and five terms (curve 5).

Acknowledgements

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