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NOISE-ENHANCED PROPAGATION OF BICHROMATIC SIGNALS

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We examine the influence of noise on the propagation of harmonic signals with two frequencies through discrete bistable media. We show that random fluctuations enhance propagation of this kind of signals for low coupling strengths, similarly to what happens with purely monochromatic signals. As a more relevant finding, we observe that the frequency being propagated with better efficiency can be selected by tuning the intensity of the noise, in such a way that for large noises the highest frequency is transmitted better than the lower one, whereas for small noises the reverse holds. Such a noise-induced frequency selection can be expected to exist for general multifrequency harmonic signals.

Keywords: Noise-enhanced propagation, stochastic resonance, bistable media, harmonic forcing, frequency multiplexing

Signal transmission is one of the most relevant processes in natural systems. Information needs to be transmitted in many different contexts, including for instance cell signalling in biological systems [1] and optical communications in technological networks [2]. Biological systems, and in particular neural tissue (where signal transmission is of utmost importance), are subjected to a large amount of noise of different origins. This fact underlies the current interest in examining the effects of random fluctuations in signal transmission processes. Actually, contrary to intuition, recent investigations have revealed the constructive role of noise in such processes. By way of example, numerical investigations have shown that random fluctuations enhance propagation of harmonic (monochromatic) signals through bistable [3] and even monostable [4] media. In those cases, the periodic response to a harmonic forcing being applied to one end of the system propagates the farthest when the amount of noise acting on all elements is optimal. The phenomenon has all ingredients characteristic of stochastic resonance [5]; one can say in fact that the system exhibits locally the noise-induced amplification of a weak periodic signal coming from the neighboring sites.

Noise-enhanced propagation has also been observed for aperiodic signals [6]. But between the two limiting cases of purely harmonic (single frequency) and completely aperiodic signals, the intermediate case of signals consisting of a finite number of harmonic modulations is worth being studied. This kind of signals is commonly used, for instance, in multichannel optical communication systems based on wavelength-division multiplexing (WDM) [2]. In a different type of application, probing methods based on the propagation of two-frequency signals are used, for example, to determine the size and abundance of plankton [7], to analyze evoked potentials in the human visual cortex [8], and to diagnose the physical conditions of the Antarctic ice sheet [9]. In this Letter, we analyze the effect of noise on the propagation of such kind of bichromatic signals. Two main conclusions can be drawn from this study. First, noise enhances propagation of the two harmonic components of the driving, similarly to what happens with standard monochromatic driving [3]. Second, and more importantly, noise can be used to select the frequency which is propagated with higher efficiency. We observe that for small noise levels the harmonic with lower frequency is propagated better than the one with higher frequency, where for large noise levels the reverse property is found. These two different effects will be analyzed in the following paragraphs.

We consider a one-dimensional chain of N coupled overdamped oscillators under the action of spatiotemporal noise. The dynamics of this system can be described by the following set of equations:

$$\dot{x}_n = f(x_n) + \varepsilon(x_{n-1} - x_n) + \varepsilon(x_{n+1} - x_n) + \xi_n(t) \tag{1}$$

where $n = 1 \dots N$ denotes the different oscillators, ε represents the strength of coupling between them, and $\xi_n(t)$ is a Gaussian noise, δ-correlated in space and time with intensity σ_a^2. The form of the deterministic force $f(x)$, which is assumed equal for all oscillators, is chosen to correspond to a bistable dynamics:

$$f(x) = k_1 x - k_2 x^3, \tag{2}$$

with k_1, $k_2 > 0$. In what follows, we will use the values $k_1 = 4.74$ and $k_2 = 7.48$. The boundary conditions of the model are such that the ends of the chain are free ($x_0 = x_1$, $x_{N+1} = x_N$). Moreover, an input signal is introduced at one end of the chain by forcing its first element with two harmonic drivings:

$$\dot{x}_1 = f(x_1) + \varepsilon(x_2 - x_1) + \xi_1(t) + A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t) \tag{3}$$

We aim to analyze the propagation of this combined signal through the chain. We choose non-commensurate frequencies and different amplitudes of the two harmonic components, to avoid interference effects due to their superposition. An example of this signal is shown in Fig. 1.

To estimate the quality of signal transmission at a certain oscillator k along the chain and at a particular frequency ω_i, we have calculated the response $Q^{(k)}(\omega_i)$ as the Fourier component of the spectrum of the corresponding time series $x_k(t)$ at this frequency:

$$Q^{(k)}(\omega_i) = \sqrt{Q_{\sin}^2 + Q_{\cos}^2}\ , \tag{4}$$

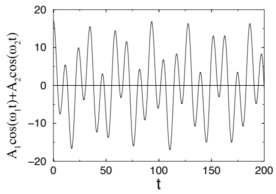

Fig 1. Two-frequency signal injected in one end of the chain. Its parameters are $A_1 = 7$, $A_2 = 10$, $\omega_1 = 0.2$, and $\omega_2 = \omega_1 \times e \approx 0.54$, where e is the base of the natural logarithm.

where

$$Q_{\sin} = \frac{\omega_i}{n\pi} \int_0^{2\pi n/\omega_i} x_k(t) \sin(\omega_i t) dt \,, \tag{5}$$

$$Q_{\cos} = \frac{\omega_i}{n\pi} \int_0^{2\pi n/\omega_i} x_k(t) \cos(\omega_i t) dt \,. \tag{6}$$

We have performed numerical simulations of model (1) in the presence of the signal shown in Fig. 1, and have analyzed the response of the different oscillators at the two driving frequencies, as a function of the intensity of the spatiotemporal noise. The results for the first oscillators of the chain are shown in Fig. 2. We first note that for the second and third oscillators (left and middle plot of the figure), the response at both frequencies decreases with noise intensity. The reason for this behavior is that the amplitudes of the two harmonic signals acting on the first oscillator are large enough to produce in it jumps between the two wells of the bistable potential even without noise. This noiseless periodic output pervades the neighboring oscillators (in this case the second and third ones). Therefore, any amount of noise decreases the quality of the response. On the other hand, far enough from the input end of the chain (depending on the value of the coupling strength; in this case, where $\varepsilon = 4$, it occurs at the fourth oscillator; for smaller coupling it occurs earlier, but the response is weaker) the system does not jump spontaneously, and noise is needed to induce the switchings between wells. For that reason, the response function of the fourth oscillator (right plot in Fig. 2) initially increases with noise intensity. Naturally, for large noise levels disorder comes into play and the response function decreases again. The result is that there exist intermediate amounts of noise for which the response at each one of the two driving frequencies is optimal, a characteristic signature of stochastic resonance. Hence, one can say that noise enhances propagation of the two-frequency signal in this system, with the optimal noise intensity being slightly different for each one of the two frequencies.

Another feature that can be observed in Fig. 2 is that the response in the first oscillators is larger at the high frequency than at the low one, for all values of the noise intensity. As the signal travels away from the input end of the chain, the response decreases monotonously, **although faster for the high frequency than for the low frequency signal. Beyond the fourth oscillator, the response is basically the same for both frequencies at zero noise, since then only intrawell motion takes place. For those oscillators, as mentioned above, a**

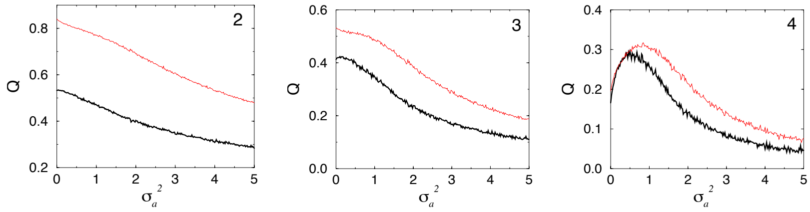

Fig 2. Response of the system in the 2nd (left), 3rd (middle), and 4th (right) oscillators at frequencies ω_1 (thick line) and ω_2 (thin line). The number of oscillators in the chain is 32.

small amount of noise induces jumps between wells, and in that situation the low-frequency response is larger (see Fig. 3), an effect which is well known to occur in standard stochastic resonance [10]. For larger noise levels the high frequency dominates again, since the system behaves in a disordered way and the larger value of A_2 prevails. As a consequence, a crossover in frequency response arises in those oscillators at an intermediate noise intensity, as shown in Fig. 3 for the fifth and sixth oscillators. The result is that, for a certain set of oscillators along the chain, noise selects the frequency which is being transmitted with better efficiency: the high-frequency harmonic for large noise intensity and the low-frequency one for small enough noise. It is worth noting that due to the intrinsic properties of the bistable chain, for equal driving amplitudes the high frequency would always be more strongly suppressed. Hence this noise-induced selection is only possible under the condition of different amplitudes.

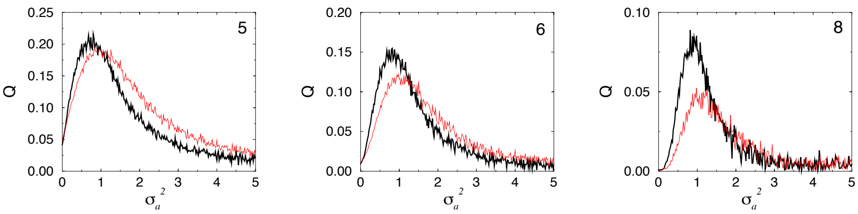

Fig 3. Response of the system in the 5th (left), 6th (middle), and 8th (right) oscillators at frequencies ω_1 (thick line) and ω_2 (thin line).

In order to visualize the noise-induced frequency selection effect described above, we have performed a symbol coding of two time series for different amounts of noise. These results are shown in Fig. 4. It can be clearly seen that for $\sigma_a^2 = 0.65$ (second plot from above) the low-frequency harmonic (top plot) is better transmitted, whereas for $\sigma_a^2 = 2.0$ (third plot from above) propagation is better for the high-frequency signal (bottom plot). The behavior of the 5th oscillator in Fig. 4 can be compared with the corresponding response curves in Fig. 3.

In conclusion, we have analyzed the constructive effect of additive noise in

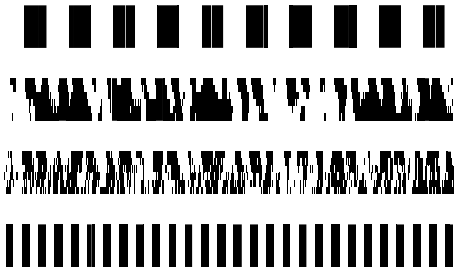

Fig 4. Symbol coding of the spatio-temporal evolution of the system. Each element is encoded in white if positive, and in black if negative. Time evolves along the horizontal axis, space along the vertical one. From top to bottom: low-frequency component of the signal ($cos(\omega_1 t)$); first six oscillators for $\sigma_a^2 = 0.65$; first six oscillators for $\sigma_a^2 = 2.0$; high-frequency component of the signal ($cos(\omega_2 t)$).

the propagation of two-frequency (bichromatic) harmonic signals through discrete bistable media. Our results show that noise enhances propagation of such signals, similarly to what happens with simpler monochromatic driving. Furthermore, we have shown that by changing the noise intensity one is able to select the propagation frequency. We expect these effects to be also present with more general multifrequency signals. Hence, this results could be relevant in biological and technological contexts where harmonic signals with many frequencies are present or used.

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References

[1] J. Keener and J. Snyder, *Mathematical Physiology*, Springer, New York (1998).

[2] G.P. Agrawal, *Fiber-Optic Communication Systems*, John Wiley & Sons, New York (1992).

[3] Y. Zhang, G. Hu, and L. Gammaitoni, *Phys. Rev. E* **58**, 2952 (1998); J.F. Lindner, S. Chandramouli, A.R. Bulsara, M. Löcher, and W.L. Ditto, *Phys. Rev. Lett.* **81** (1998) 5048-5051.

[4] A.A. Zaikin, J. García-Ojalvo, L. Schimansky-Geier, and J. Kurths, *Phys. Rev. Lett.* **88** (2002) 010601.

[5] K. Wiesenfeld and F. Moss, *Nature* **373** (1995) 33; A.R. Bulsara and L. Gammaitoni, *Phys. Today* **49** (1996) 39; L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, *Rev. Mod. Phys.* **70** (1998) 223-287.

[6] J. García-Ojalvo, A.M. Lacasta, F. Sagués, and J.M. Sancho, *Europhys. Lett.* **50** (2000) 427-433.

[7] R.B. Mitson, Y. Simard, and C. Goss, *ICES J. Marine Science* **53** (1996) 209-215.

[8] J.D. Victor and M.M. Conte, *Visual Neuroscience* **17** (2000) 959-973.

[9] S. Fujita, H. Maeno, S. Uratsuka, T. Furukawa, S. Mae, Y. Fujii, and O. Watanabe, *J. Geophys. Res.* **104** (1999) 13013-13024.

[10] V.S. Anishchenko, A.B. Neiman, F. Moss, and L. Schimansky-Geier, *Physics-Uspekhi* **42**, 7 (1999).