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SELF-EXCITATION OF IMPINGING JETS WITH REGARD  
TO ACOUSTIC FEEDBACK

A.S.Ginevsky, P.S.Landa, and A.A.Zaikin

\* Central Aerohydrodynamics Institute (TSAGI), Moscow, Russia  
\*\* Moscow State University, Department of Physics, Moscow, Russia

ABSTRACT

The traditional point of view is based on the idea of existence of discrete vortices - coherent structures - and their near and far interactions. Since the vortex interaction is an essentially non-linear process, all the observed effects from this standpoint can be explained only in terms of the non-linear theory. Meanwhile, the nature of the examined processes is the instability of jet mixing layer (instability of Kelvin-Helmholtz). The instability notion itself, in the sense of small disturbance increase, is linear. Therefore it is naturally to expect that many results associated with the mechanism of coherent structure self-excitation in free jets and of self-oscillations in jets with obstacles can be explained in the frames of the linear theory.

If a jet impinges on a flat plate powerful self-oscillations appear for sufficiently large subsonic velocities of flow, when the Mach number  $M > 0.6$ , and for sufficiently small distances from the nozzle exit to flat plate ( $x_0/d < 7.5$ , where  $x_0$  is the distance from the nozzle exit,  $d$  is diameter of the nozzle). The frequency of the self-oscillations depends on the distance from the nozzle exit to flat plate but it falls in the certain range adjacent to the frequency of maximum of the pulsation power spectrum in the end of initial part of the free jet [1-4]. The excitation of these self-oscillations is because the appearance of feedback via acoustic wave propagating against the flow. This wave appears due to impinging of the ring vortices on the flat plate. For determination of the excitation conditions of these self-oscillations the authors of the majority of works (see for example [2]) use the resonance condition of thumb having the form

$$x_0/\lambda_h + x_0/\lambda_a = N, \quad (1)$$

where  $\lambda_h$  and  $\lambda_a$  are the lengths of hydrodynamical and acoustic waves

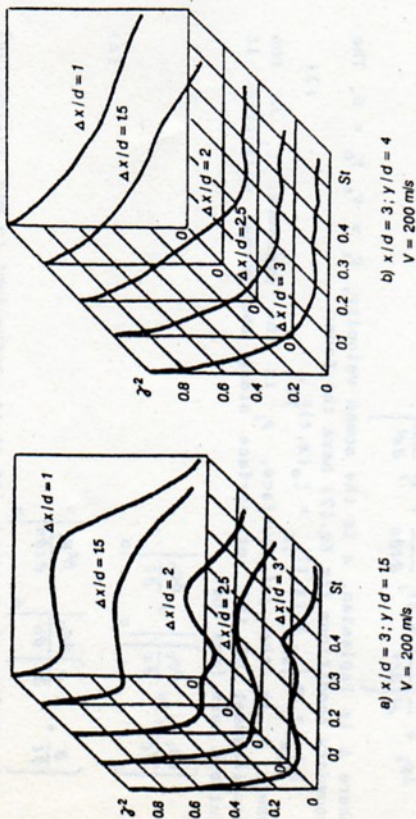


Fig. 4 Coherence function of pressure pulsations when the microphones being moved apart longitudinally

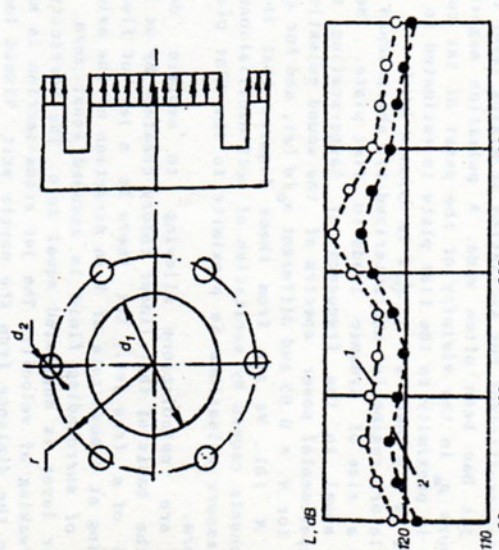


Fig. 5 Noise spectra of a full scale jet in the near field  
( $Re = 5.8 \cdot 10^6$ ,  $T = 870 K$ ,  $M = 1$ )

1 - reference jet  
2 - jet with peripheral jets

respectively,  $N$  is integer.

To account for such fact as impossibility of excitation of the self-oscillations for small velocities ( $M < 0.6$ ) in spite of feedback via the acoustic wave, the assumption that intensity of the acoustic wave is insufficient for excitation of mixing layer close to the onset of jet has been often made. A pulsation magnitude of acoustic pressure  $P_a$  in the vicinity of the onset of jet caused by turbulent flow in proximity to the flat plate is estimated in [4]. It is proportional to  $Mq$  where  $q = \rho_0 \omega_0^2 / 2$  is dynamic head.

Self-oscillatory regime is characterized by sharp sound in far jet field and a rise of dynamic loads to flat plate. The sound frequency is equal to the frequency of large-scaling coherent structures. Experimental power spectra of the sound pulsations are shown in Fig. 1 for  $M = 0.95$  and different  $x_0/d$  (a), and for  $x_0/d = 4$  and different  $M$  (b). We see from these figures that there are discrete components caused by excitation of self-oscillations. Power spectra of pressure pulsations in proximity to the flat plate have the similar form.

Following are calculations allowing to explain described phenomena on the basis of the linear theory created by us for the simplest model of a free jet. Let there is a jet of fluid with density  $\rho$  moving at a mean rate of  $V$  in direction of the axis  $x$ . The mean velocity of surrounding fluid is assumed equal zero. So, the width of shear layer is suggested equal zero. The vorticity occurs due to the breaking of velocity. The jet cross-section is suggested independent on the distance from the nozzle exit. Viscid losses are taken into account in the final results.

The linearized equations of hydrodynamics for inviscid compressible fluid are used as starting ones. Going in these equations to velocity potentials  $\varphi_j$  inside ( $j = 1$ ) and outside ( $j = 0$ ) of the jet we obtain

$$\Delta \varphi_j = \frac{1}{a^2} \frac{\partial^2 \varphi_j}{\partial t^2} + 2V_j \frac{\partial^2 \varphi_j}{\partial t \partial x} + V_j^2 \frac{\partial^2 \varphi_j}{\partial x^2}, \quad (2)$$

where  $\Delta$  is laplacian,  $a$  is the sound velocity,  $V_1 = V$ ,  $V_0 = 0$ . The boundary conditions for Eq. (2) have the form:

$$P_1|_S = P_0|_S, \quad \zeta_1(x, t)|_S = \zeta_0(x, t)|_S, \quad (3)$$

where  $S$  is the jet surface,  $P_j$  is the pressure,  $\zeta_j$  is the displacement of the jet surface along the normal direction. It follows here from that

$$\left[ \frac{\partial \varphi_1}{\partial t} + V \frac{\partial \varphi_1}{\partial x} \right]_S = \frac{\partial \varphi_0}{\partial t} \Big|_S, \quad (4)$$

$$\left[ \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right] \frac{\partial \varphi_0}{\partial n} \Big|_S = \frac{\partial^2 \varphi_1}{\partial t \partial n} \Big|_S$$

In studies of a round jet it is convenient to use cylindrical

coordinates  $x$ ,  $r$  and  $\vartheta$ . In this coordinates the solution of Eq. (2) has the form:  $\varphi_j(x, r, \vartheta) = \tilde{\varphi}_j(r, \vartheta) e^{i(\omega t - kx)}$ , where  $\tilde{\varphi}_j(r, \vartheta)$  is a solution of the equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial \tilde{\varphi}_j}{\partial r} \right] + \frac{1}{r^2} \frac{\partial^2 \tilde{\varphi}_j}{\partial \vartheta^2} = \kappa_j^2 \tilde{\varphi}_j. \quad (5)$$

Here  $\kappa_j = k^2 - \frac{(\omega - kV_j)^2}{a^2}$ . Eqs. (5) can be solved by the variable separation method; to do this, we put  $\tilde{\varphi}_j = R_j(r) \Theta_j(\vartheta)$ . It follows from Eq. (5) that  $\Theta_j(\vartheta) = \cos n\vartheta$ , and  $R_j(r)$  is determined by the Bessel equation

$$r \frac{d}{dr} \left[ r \frac{dR_j}{dr} \right] = \kappa_j^2 r^2 R_j + n^2 R_j \quad (6)$$

If the solution of Eq. (6) is to be limited at  $r = 0$  and tended to zero at  $r \rightarrow \infty$ , we put

$$R_j = A I_n(\kappa_j r), \quad R_0 = B K_n(\kappa_0 r), \quad (7)$$

where  $I_n(\kappa_j r)$  is the Bessel function of imaginary argument,  $K_n(\kappa_0 r)$  is the Macdonald function. The system of equations for unknown constants  $A$  and  $B$  follows from the boundary conditions (4). Setting equal to zero the determinant of this system, we obtain the dispersive equation

$$\left( \omega - kV \right)^2 \kappa_0 I_n(\kappa_1 R) \left[ K_{n-1}(\kappa_0 R) + \frac{n}{\kappa_0 R} K_n(\kappa_0 R) \right] + \omega^2 \kappa_1 K_n(\kappa_0 R) \left[ I_{n-1}(\kappa_1 R) - \frac{n}{\kappa_1 R} I_n(\kappa_1 R) \right] = 0, \quad (8)$$

where  $R$  is the jet radius.

It is follows from the dispersive equation (8) that the integer  $n$  has to be even. Dependencies of real and imaginary parts of roots of the dispersive equation (8) on the Strouhal number for hydrodynamical and acoustic waves are shown in Fig. 2. We see that both real and imaginary part of hydrodynamical wave number increase monotonically with rise of  $St$ . Monotonous increase of spatial gain factor with rise of the Strouhal number is contrary to experimental data. This increase is result from the fact that the width of shear layer in our calculations was suggested equal zero. Taking into account a finite width of shear layer, i.e. in essence viscid losses, Michalke [5] has obtained a finite Strouhal number corresponding to a maximum in amplification of hydrodynamical wave. If we will suggest the viscid losses to be proportional to the frequency squared then we may describe following expression for the spatial gain factor of hydrodynamical perturbations with the Strouhal number  $St$ :

$$\alpha_h(S,t) = \text{Im}(k_h R) - \frac{\delta(x)}{R} S t^2, \quad (9)$$

where  $\delta(x) = b + ax$  is a quantity proportional to the width of shear layer. Coefficients  $b$  and  $a$  can be estimated from known experimental data. Dependence  $\alpha_h(S,t)$  for  $x/R \approx 10$ ,  $b = 2$ ,  $a = 0.28$  is shown in Fig. 2 too. We see that maximum of the gain factor is accounted for by  $S t \approx 0.33$ , as it is observed in experiment.

Because excitation of self-oscillations for impinging jet takes place due to interaction between hydrodynamical and acoustic waves, we must impose two boundary conditions for determination of self-excitation condition. The boundary condition on the flat plate can be obtained by recognizing that total momentum imparted to the plate by hydrodynamical wave is partially transferred to the momentum obtained from the plate by acoustic wave. So, we have

$$\int_{-\infty}^{\infty} (R_1 u_h + u_a) \Big|_{x=x_0} dx = 0, \quad (10)$$

where  $u_h$  is component of hydrodynamical velocity along the axis  $x$ ,  $u_a$  is corresponding component of acoustic velocity,  $R_1$  is a factor of the transformation of hydrodynamical wave into acoustic one on the flat plate. Similarly, the boundary condition on the nozzle exit we can describe as

$$u_h(0, R, t) + u_h(0, R, t) + R_2 [u_a(0, R, t) + u_a(0, R, t)] = 0, \quad (11)$$

where  $R_2$  is a factor of the transformation of acoustic wave into hydrodynamical one on the nozzle exit. Let us note that the factors  $R_1$  and  $R_2$  can be complex and depending on frequency.

(11), we get a system of equations for unknown amplitudes of hydrodynamical and acoustic waves. Setting determinant of this system equal to zero, we obtain following characteristic equation for complex frequency  $\omega$ :

$$e^{i(k_h + k_a)x_0} x_0 = F, \quad (12)$$

where

$$F = R_1 R_2 \frac{\mathcal{F}(k_h)}{\mathcal{F}(-k_a)}, \quad (13)$$

$$\mathcal{F}(k) = \frac{1}{\pi S t - k R / 2} \int_0^R \int_0^{\infty} \frac{1}{1 - \frac{k R}{\pi S t}} \frac{1}{K_0(x_0 R)} K_0(x_0 r) dr.$$

Taking into account the viscous losses, we change in the left side of Eq. (12) from  $\text{Im}(k_h R)$  to  $\alpha_h(S,t)$  which is determined by formula (9). In so doing Eq. (12) can be described in the form of system of following equations:

$$\exp \left[ \left[ -\text{Im}(k_h R) + \frac{b + a x_0}{R} (\text{Re} S t)^2 \right] \frac{x_0}{R} \right] = |F|, \quad (14)$$

$$\tan \left[ \text{Re}(k_h + k_a) x_0 \right] = \frac{\text{Im} F}{\text{Re} F}.$$

Eqs. (14), in combination with solutions of Eqs. (8) for hydrodynamical and acoustic wave numbers, determine frequencies and increments of excited self-oscillations for any given values of  $M$  and  $x_0$ . Computations have shown that the imaginary part of  $F$  is small in comparison with its real part for real values of  $R_1 R_2$ . So, we can put

$$\text{Re}(k_h + k_a) x_0 \approx 2\pi N, \quad (15)$$

where  $N$  is integer. It is easily seen that the condition (15) coincides with the resonance condition of thumb (1). The results of computations of Eqs. (14) are demonstrated in Fig. 3 for  $R_1 R_2 = 1$  and  $M = 0.9$ . Dependencies of increment  $\Delta = -\text{Im} S$  on relative distance  $x_0/d$  between the nozzle exit and the flat plate are shown in Fig. 3.

Corresponding dependencies of the frequency of self-oscillations, which is proportional to the real part of the Strouhal number, are shown in Fig. 3. It is seen from these figures that for the same value  $x_0$  the excitation conditions can be fulfilled for several modes simultaneously. It is evident that the mode with the largest increment will most likely be excited. However, hysteresis has to take place when the distance from the nozzle exit to the plate varies in opposite directions. This hysteresis has been observed in experiment [2]. The dependencies of the frequencies of the modes with the largest increment on  $x_0/d$  are shown in Fig. 3 by solid lines. It follows from this figure that the self-oscillations are excited if the distance from the nozzle exit to the plate is not more than certain critical value depending on the Mach number. The Strouhal numbers for these self-oscillations are embedded in certain interval. As the distance from the nozzle exit to the plate increases, the frequency of the self-oscillations smoothly reduces to certain value and then increases by jump. It means that the jump from one mode to another takes place. Furthermore, similar changes are repeated again. The dependencies of the frequencies of the excited self-oscillations on the distance between the nozzle and the plate calculated by us are in full agreement with experiment [2]. Our calculations have shown that self-excitation is possible starting from some critical Mach number only. It is in full agreement with experiment too. Hence we have shown that the phenomena, which in the majority of works are explained by vortex pairing, i.e. by nonlinearity, are in fact linear ones.

Let us note that the self-excitation conditions and values of increments of self-oscillations essentially depend on  $R_1 R_2$ , i.e. on properties of the flat plate and the nozzle exit. This can be used for control by intensity of self-oscillations [1, 6].

The developed theory allows to calculate the self-excitation

conditions and frequencies of self-oscillations for jets impinging on a wedge, for open-jet return-circuit wind tunnels and so on.

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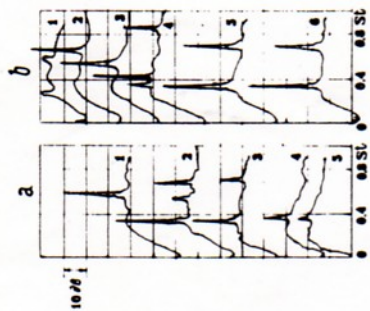
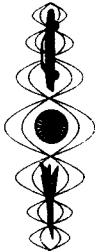


Fig. 1. Experimental pressure fluctuation spectra in the far acoustic field of the impinging jet.

- a -  $M = 0.95$  ( $1 - x_0/d = 2$ ,  
 2 - 4, 3 - 6  
 4 - 8, 5 -  $x_0/d = 9$ );  
 b -  $x_0/d = 4$  ( $1 - M = 0.63$ ,  
 2 - 0.73, 3 - 0.77,  
 4 - 0.87, 5 - 0.92,  
 6 -  $M = 0.95$ ).



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ACOUSTIC CHARACTERISTICS OF CONFINED JETS

Kam W. Ng  
Office of Naval Research  
Arlington, VA 22217-5660  
U.S.A

ABSTRACT

An experimental study was conducted to investigate the flow-induced noise and vibration caused by confined jet flows in a cylindrical duct. Unrestricted pipe flow and flows restricted by various orifices were tested for a wide range of velocities to simulate the confined jet flow in piping systems. Wall pressure data showed that the noise levels vary with the pipe's axial location, and the peak noise is located near the end of the jet potential core. Scaling of noise with jet velocity indicated that the value of velocity exponent decreases with increasing jet diameter. A non-dimensional wall pressure spectrum was established for the various confined jets by the Strouhal relationship, where the length scale is the jet hydraulic diameter. This jet pressure spectrum agrees well with the wall pressure spectrum of a turbulent boundary layer above a rigid plane. Correlations of wall pressure fluctuations and pipe wall acceleration signals showed that jet flows generate more deterministic features than pipe flow. The coherent functions of the wall pressure and pipe wall acceleration signals are relatively high near the exit of the jet. The high coherence is due to the large-scale coherent structures.

INTRODUCTION

Turbulence-generated noise caused by high velocity flow through valves and restrictors has been identified as one of the major noise contributors in piping systems. To predict and reduce piping system noise, a better understanding of the flow field around the flow restriction and its noise generating mechanisms is needed. The noise characteristics of flow through valves and restrictors with air, steam, and gases have been studied quite extensively.<sup>1-5</sup> However, studies of the noise characteristics of water flow through valves and regulators have been limited, and the understanding of flow-induced noise is far from satisfactory.

The physical problem under investigation is depicted in Fig. 1. In general, a low-velocity flow approaches the flow restriction in a pipe. A high-velocity jet is formed at the vena contracta immediately downstream of the orifice plate. Near the orifice plate, the jet is surrounded by a low velocity recirculation zone or reversed flow region. Further downstream, the jet shear layer grows until it attaches to the pipe wall. As shown in the figure, the flow field can be divided into two regions; namely, the recirculating or developing flow region and the fully developed region. The developing region is the noise production area in which the flow and noise

Fig. 3. Dependences  $Im$ s and  $Re$ s on the relative distance  $x_0/d$  for  $M=0.9$ .

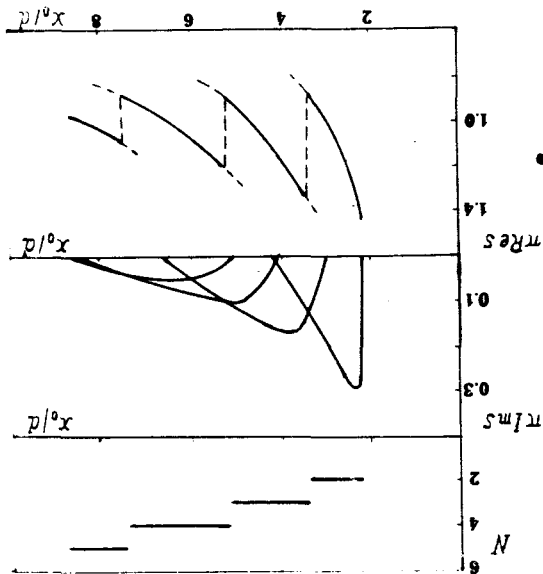


Fig. 2. Dependences of real and imaginary parts of roots of the dispersive equation (8) on the Strouhal number for round free jet.  $M=0.95$ .  
1, 1' -  $M=0.25$ , 2' -  $M=0.5$ , 3' -  $M=0.75$ , 4, 4' -

