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ACOUSTIC NONLINEARITY OF NONUNIFORM FLOW OF AN OSCILLATING LIQUID

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The efficiency of nonlinearity production by a localized geometrical inhomogeneity is estimated using as an example of the latter a particle layer. The Bernoulli equation for the near field and a Lighthill-type equation in the wave zone are employed in the calculation. Use is also made of the equivalent nonlinearity parameter.

In an attempt to improve the efficiency of wave interactions one should search for new nonlinear materials and also try to make better use of the potentialities of the ordinary nonlinear mechanisms. Recently intensive studies have been carried out of structurally inhomogeneous media (liquids with gas bubbles and porous or defective solid-state specimens) whose nonlinearity parameter ε attains the values of 10^2–10^4. At the same time, the total contribution of the physical and geometrical nonlinearities of these media (relating, respectively, to the nonlinear dependence of intermolecular interaction forces on deformation and to the presence of nonlinear terms in the equations of continuum mechanics) usually does not exceed 10^1.

The role of geometrical nonlinearity can be noticeably increased. Consider, for example, the convective nonlinear term in the equation of motion whose contribution is characterized by the number

$$M = |(\mathbf{u}\nabla)\mathbf{u}|/|\partial\mathbf{u}/\partial t| \sim u_0 T/L. \tag{1}$$

Here u_0 is the characteristic velocity of the flow and T and L are its time and space scales, respectively. For a sound wave the parameter T is the period and $L = c_0 T = \lambda$ is the wavelength (c_0 is the velocity of sound), and therefore $M = u_0/c_0$ is the acoustic Mach number, which does not exceed 10^{-4}–10^{-2} even in very strong fields.

If an obstacle of dimension $R \ll \lambda$ is placed in a low-frequency field, the scale of the flow near the obstacle will be $L \sim R$, and the number M will increase λ/R times. Hence, by placing small obstacles in an oscillating liquid one can strengthen the nonlinearity by means of purposeful formation of small-scale inhomogeneities of the flow.

As is known, maximum gradients are produced in an acoustic boundary layer of thickness $\delta = (\nu/\pi f)^{1/2}$, where ν is the kinematic viscosity and f is frequency. Thus, the local increase of the nonlinearity parameter is estimated as $\lambda/\delta \sim c_0(\pi/\nu f)^{1/2}$. In particular, for air at frequencies $f \sim 100$ Hz, the nonlinearity increases 10^4 times.

In many experiments, when a hole in a screen was irradiated by intensive sound certain nonlinear effects were observed: changes of the impedance with increasing wave amplitude [1] and generation of harmonics [2]. Allowance for the geometrical nonlinearity is important in calculations of resonance absorbers based on the Helmholtz resonator and operating at high sound levels [3]. Radiation from an oscillating sphere was calculated in [4], but the medium considered there had ordinary types of nonlinearity.

Consider a flow nonuniformity produced by solid spherical particles that are not entrained by the flow and are concentrated in a thin layer of thickness $d \ll \lambda$. The volume concentration n_v is assumed to be low so that the flow perturbation produced by a single particle should not actually affect the flow around the neighboring particles. Let the flow be created by a plane acoustic wave $\mathbf{u} = \mathbf{u}_0 \cos(\omega t - kz)$ normally incident on the layer.

It is well known that when the obstacle has small wave dimensions the compressibility can be neglected. The solution to the problem of potential flowing around the sphere has the form [5]

$$\mathbf{v} = \mathbf{u} - \frac{R^3}{2r^3}\left[\frac{3}{r^2}\mathbf{r}(\mathbf{u}\mathbf{r}) - \mathbf{u}\right]. \tag{2}$$

Here R is the radius of the spherical particle and $\mathbf{r}$ is the radius vector from the center of the sphere to the observation point. The distribution of pressure is given by the formula

$$p = p_0 - \frac{1}{2}\rho v^2 - \rho\frac{\partial\varphi}{\partial t}, \tag{3}$$

where φ is the potential ($\mathbf{v} = \nabla\varphi$), ρ is the density of the liquid, and p_0 is the pressure in the unperturbed flow.

Using the solutions (2), (3) and separating out the Fourier component of the pressure at the frequency of the second harmonic, we obtain

$$p_{2\omega} = \frac{1}{4}\rho u_0^2\left[\frac{R^3}{r^3}(3\cos^2\theta - 1) - \frac{1}{4}\frac{R^6}{r^6}(3\cos^2\theta + 1)\right]. \tag{4}$$

Here θ is the polar angle between $\mathbf{r}$ and the z axis.

In the estimation we take the amplitude of the second harmonic on the axis at a distance $d \sim nR$ from the center of the spherical particle as a characteristic pressure:

$$p_{2\omega} \approx 0.5\rho u_0^2 n^{-3}. \tag{5}$$

Now we compare this with a different situation. Let a sound wave be incident on a layer of a homogeneous nonlinear medium of thickness d. According to the Bessel–Fubini solution [6], the amplitude of the second harmonic at the exit from the layer is

$$p_{2\omega} = \rho c_0 u_0 J_2\left(\frac{2\varepsilon}{c_0^2}\omega u_0 d\right)\Big/\left(\frac{\varepsilon}{c_0^2}\omega u_0 d\right) \approx \frac{\varepsilon}{2c_0}\rho\omega u_0^2 d. \tag{6}$$

Here ε is the nonlinearity parameter of the medium and J_2 is Bessel's function. Comparing expressions (5) and (6) and using (6) as a definition of the equivalent nonlinearity parameter, we obtain for the particle layer

$$\varepsilon = \frac{c_0}{\omega d n^3} = \frac{\lambda}{2\pi d n^3} \approx \frac{1}{2\pi}\frac{\lambda}{d}\left(\frac{R}{d}\right)^3. \tag{7}$$

Formula (7) implies that since $\lambda \gg d$, in the consideration of the near field the equivalent nonlinearity parameter satisfies the condition $\varepsilon \gg 1$. Consequently, such a system can efficiently generate oscillations of the second harmonic (and also combination frequencies if the layer is exposed to a biharmonic signal).

Now we proceed to analysis of the distant field. Let us use a Lighthill-type equation [5]:

$$\Delta P - \frac{1}{c_0^2}\frac{\partial^2 P}{\partial t^2} = -\frac{1}{2}\frac{\rho}{c_0^2}\frac{\partial^2 \mathbf{v}^2}{\partial t^2}, \tag{8}$$

where the potential character of the flow is taken into account and the right-hand side is nonzero only in the vicinity of the inhomogeneity region and describes sources exciting the second harmonic. In Eq. (8) we have $P = p' + \rho\mathbf{v}^2/2$, and in the distant zone the variable P coincides with the acoustic pressure p'.

Solving (8) by the method of retarded potentials, for the second harmonic wave produced by flowing around a single spherical particle we obtain

$$P(\mathbf{r}, t) = -P_{2\omega}\frac{1}{r}\cos\left(2\omega t - \frac{1}{c_0}2\omega r\right), \qquad P_{2\omega} = \frac{\rho\omega^2 u_0^2 R^3}{6c_0^2}. \tag{9}$$

Now we add together the perturbations from all particles assuming that the layer is so thin that the inhomogeneities of the oscillating flow in the vicinity of each of the spherical particles radiate in phase. The amplitude of the resultant field is

$$p_{2\omega} = \left(\frac{\pi}{3}R^3 n_v\right)\left(\frac{1}{2c_0}\omega u_0^2\rho d\right). \tag{10}$$

Comparing (10) and (16) we obtain an estimate for the nonlinearity parameter:

$$\varepsilon \approx \frac{\pi}{3} R^3 n_v \approx \frac{R^3}{a^3} < 1, \tag{11}$$

where a is the average distance between the particles. Thus, the geometrical nonlinearity produced by introducing small inhomogeneities in an oscillating flow and exerting a strong effect near the obstacle, turns out to be small in the wave zone.

Amplification of nonlinear interactions can be achieved in experiments. To this end it is convenient to introduce geometrical inhomogeneities produced by systems of similar elements, for instance, by a layer of wires or a stack of plates aligned normally to the wave. The first of these problems is solved by analogy with what was presented above, and to solve the other problem one must solve the Prandtl equation for the boundary layer.

Let us discuss the field of applicability of the above approximations. The neglect of dissipative processes is equivalent to the employment of the condition of a large Reynolds number Re. Using the expression $\mathrm{Re} = u\rho R/\eta$, where η is the dynamic viscosity, the definition of the sound pressure level $N = 20 \log(p/p_0)$, where $p_0 = 2 \times 10^{-4}$ dyn/cm^2, and also the relation $p = \rho c_0 u$, we can derive the expression

$$\mathrm{Re} = \frac{R}{\eta c_0} p_0 \times 10^{N/20}. \tag{12}$$

Formula (12) implies that the condition $\mathrm{Re} > 1$ is attained at sound pressure levels $N > 110$ dB (for a spherical particle with $R = 0.1$ cm) and $N > 90$ dB (for a sphere with $R = 1$ cm). In the problem under consideration the tangential velocity component on the surface of the spherical particle had a finite value, whereas in a real viscous medium at large Reynolds numbers the velocity drops down to zero in a thin near-wall liquid layer. Thus, the calculations are correct provided the thickness of the boundary layer satisfies the condition $\delta = (\nu/\pi f)^{1/2} \ll R$.

This condition gives $f \geqslant 500$ Hz for a spherical particle of radius $R = 0.1$ cm and $f \geqslant 5$ Hz for $R = 1$ cm.

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