Additive Noise and Noise-induced Nonequilibrium Phase Transitions

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Abstract. We study how additive noise influences noise-induced nonequilibrium phase transitions. The review of the most recent results and a discussion about open questions is presented. We show that additive noise strongly influences a noise-induced phase transition. We demonstrate also that additive noise can induce such a transition by itself. It can occur both via the excitation of noise-induced oscillations and via the creation of a mean field in the nonlinear systems. Finally, we show that additive noise can stabilize noise-induced oscillations.

INTRODUCTION

Nonlinear systems which demonstrate noise-induced order have been in the centre of attention over the past two decades. There have been discovered several interesting phenomena as *noise-induced nonequilibrium phase transitions* [1–4], *stochastic resonance* [5–7], *noise-induced transport* [8], *coherence resonance* [9] or *noise-induced pattern formation* [10].

Discovered in 1980-s (see for a review [1]) *noise-induced transitions* have been confirmed by numerous experimental works (e.g. [11]). In early 1990-s an interest has been again attracted to noise-induced transitions, now in connection with *noise-induced phase transitions.* The key feature of these transitions is that the shape of the probability distribution of the coordinate or amplitude squared is not qualitatively changed in the course of the phase transition and that the noise does not induce maxima in this probability distribution in contrast to transitions described in [1]. Noise-induced phase transitions have been detected in distributed systems [2,3], where a transition resulted in breaking a symmetry and in the creation of a mean field, as well as in nonlinear oscillators [12–14], where a transition manifested in the excitation of noise-induced oscillations. The noise intensity in such transitions plays the role of the temperature and the order parameter is represented by the mean field in non-oscillatory systems or the averaged instantaneous amplitude in oscillatory systems. It is also important to note that these studies have lead to the treatment of the following topics: a control of noise-induced oscillations [15],

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the on-off intermittency resulted from noise-induced transitions [16] and first-order phase transitions [17].

In the majority of works about noise-induced phase transitions multiplicative noise is perceived to be responsible for such transitions. However, recent results [18–20] have shown that additive noise can also be of a crucial importance in such phase transitions. We note that an action of an additive noise is qualitatively different from multiplicative ones: additive noise acts on the system as a force, whereas multiplicative noise performs parametrical action. We have recently shown that additive noise can crucially influence a noise-induced phase transition [18,20,21] and can even induce such a transition [22,20].

In the present paper we review several of these aspects of additive noise and noise-induced nonequlibrium phase transitions. Studying several examples we discuss three types of noise influence: a noise-induced phase transition in the presence of additive noise, a hidden phase transition induced by additive noise and a stabilization of oscillations, performed by additive noise. Finally, we summarize results obtained and discuss open questions and possible directions of the future investigations.

TRANSITIONS IN THE PRESENCE OF ADDITIVE NOISE

To show the influence of additive noise on the transition induced by multiplicative noise, let us consider a pendulum with suspension axis randomly vibrated in a certain direction making an angle γ with the vertical axis [12,18,20]. The equation of motion for this system can be written as

$$\ddot{\varphi} + 2\beta(1 + \alpha\dot{\varphi}^2)\dot{\varphi} + \omega_0^2 \sin\varphi + \omega_0^2 \xi(t) \sin(\varphi + \gamma) = 0, \tag{1}$$

where φ is the pendulum angular deviation from the equilibrium position, ω_0^2 is the natural frequency of small free pendulum oscillations, β is a damping factor with the nonlinear coefficient α, $\xi(t)$ is a comparatively wide-band random process.

Assuming that an intensity of the suspension axis vibration is moderately small, the equation of motion becomes

$$\ddot{\varphi} + 2\beta(1 + \alpha\dot{\varphi}^2)\dot{\varphi} + \omega_0^2(1 + \xi_1(t))\varphi = \omega_0^2 \xi_2(t) \tag{2}$$

where $\xi_1(t) = \xi(t)\cos\gamma$ is a multiplicative component of the suspension vibration, and $\xi_2(t) = -\xi(t)\sin\gamma$ is its additive component.

Now let us analyze what happens with the increase of the multiplicative noise intensity. Firstly, we analyze the case without additive noise ($\xi_2 = 0$). Using the Krylov-Bogolyubov method to obtain equations for the amplitude and phase, and solving these equations by a method of the Fokker-Planck equation, one can show that the system undergoes a noise-induced transition. The dependence of the averaged amplitude squared $a\langle A^2\rangle$ on η, where $a = 3\alpha\omega_0^2/4$, is shown by curve 1 in Fig. 1, *left.* The parameter $\eta = 3\omega_0^2(k - k_{cr})/16\beta$ is the extent to which the intensity of multiplicative noise κ exceeds its critical value κ_{cr}. This figure shows that a parametrical action of multiplicative noise leads to the phase transition: below a threshold we get $\langle A^2 \rangle = 0$, and after a threshold noise-induced oscillations are excited and its intensity increases with an increase of the noise intensity. This transition has been also studied analytically in [12]. Note, that after a transition no maxima are induced in the probability distribution for the amplitude squared [19].

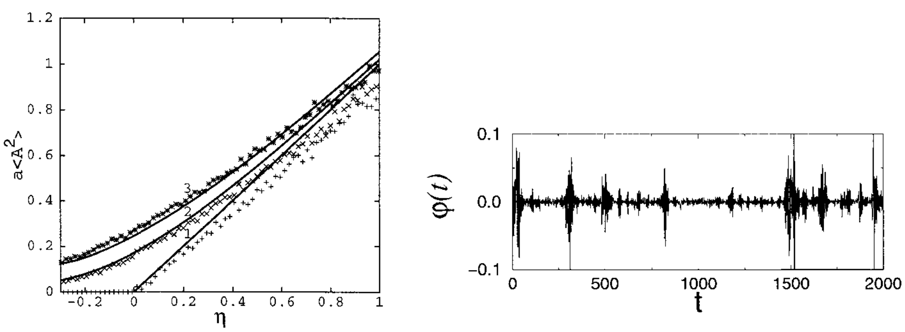

FIGURE 1. *Left:* A noise-induced phase transition in a pendulum with randomly vibrated suspension axis. The dependence of a value proportional to the averaged amplitude squared on η, where η is an extent on which multiplicative noise intensity exceeds the threshold value. The curve 1 corresponds to the case without additive noise, curves 2 and 3 to the cases with additive noise intensities $k1$ and $k2$, where $k2 > k1$ (for details and analytical expressions see [20]). Analytical and numerical results are shown by solid and symbol curves, respectively. *Right:* On-off intermittency for subcritical values of multiplicative noise intensity.

With an increase of additive noise intensity the picture of the transition will be qualitatively changed (curves 2 and 3 in Fig. 1, *left*, correspond to the additive noise intensities $\kappa 1$ and $\kappa 2$, where $\kappa 2 > \kappa 1$). A presence of additive noise leads to the fact that a probability distribution below the threshold is not longer a δ-function, and the transition is now not so well-defined, as in the case without additive noise. But again the probability distribution is not qualitatively changed after the transition [19].

The additive noise also influences the on-off intermittency [16,20]. Without additive noise on-off intermittency is observed near a threshold for supercritical values of multiplicative noise intensity, whereas in the presence of additive noise this on-off intermittency is hidden, but now observable below a threshold (see Fig. 1, *right*).

So, we have discussed a phase transition induced by multiplicative noise and have shown that additive noise can hide this transition and make it very difficult for a detection.

TRANSITIONS INDUCED BY ADDITIVE NOISE

Transitions in nonlinear oscillators

The model considered in the previous section can serve as a key model for understanding another effect, a hidden phase transition induced by additive noise. One can show it by a consideration of an oscillator with a quadratic nonlinearity and additive noise [20]. The equation of the oscillator is:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2(1 + x + \gamma x^2)x = \omega_0^2 b\xi(t). \tag{3}$$

where β is a friction, ω_0 is a natural frequency , and b is a parameter responsible for the intensity of additive noise $\xi(t)$.

We note that the system under consideration belongs to the class of systems which may demonstrate a transition without qualitative changes in the probability distribution. Also it is obvious that additive noise will make every transition hidden (see the previous section). Therefore, despite the presence of the transition mechanism the direct use of Fokker-Planck equation and its solution does not show a transition. We call this phenomenon as *hidden noise-induced phase transition.*

To reveal the mechanism of the hidden transition induced by additive noise, let us decompose the initial variable x into

$$x(t) = y(t) + \chi(t), \tag{4}$$

where $\chi(t)$ is a filtered additive noise satisfying

$$\ddot{\chi} + 2\beta\dot{\chi} + \omega_0^2\chi = \omega_0^2 b\xi(t). \tag{5}$$

Using this decomposition for y we obtain:

$$\ddot{y} + 2\beta\dot{y} + \omega_0^2\Big(1 + y + \xi_2(t) + \gamma y\big(y + 3\chi(t)\big)\Big)y = \omega_0^2\xi_1(t), \tag{6}$$

where $\xi_1(t) = -\chi^2(t)\big(1 + \gamma\chi(t)\big)$ is additive noise and $\xi_2(t) = \chi(t)\big(2 + 3\gamma\chi(t)\big)$ is multiplicative noise.

The solution of the eq.(6) with $\xi_1(t) = 0$ is denoted as y_r. Comparing the eq.(6) with (2), it is clearly seen that these eqs. are very similar. Taking this into account, it is not surprising now that this equation demonstrates a clearly-defined transition in the absence of additive noise (see Fig. 2, *left*, (a), curve 2). A presence of additive noise makes the transition hidden (the same figure, curve 1 and 3).

If additive noise has no spectral components on the resonant frequency, one can drop the term $3\gamma\chi^2$ from eq.(6) and confirm the transition by an analytical calculation [20]. In this case we denote the solution of eq.(6) as y_{rr} (in this case also $\xi_1(t) = 0$). The dependence of oscillation variance on noise intensity then looks as curve 4 in the same figure. Instead of a variance, mean values can be used as an order parameter for the treatment of the transition (see Fig. 2, *left*, (b)).

So, we have shown that an oscillator with quadratic nonlinearity and additive noise may demonstrate a hidden noise-induced transition. The physical mechanism of this effect is an autoparametrical excitation.

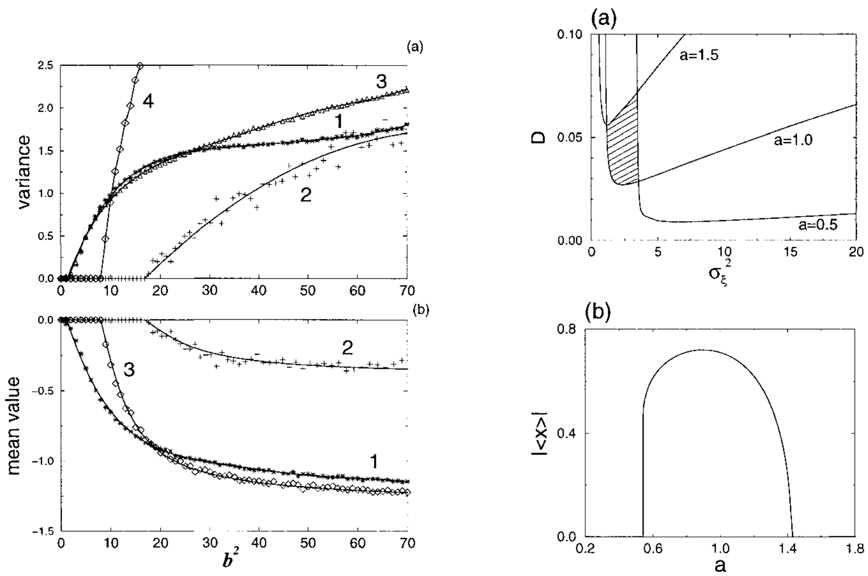

FIGURE 2. *Left:* The hidden transition in a nonlinear oscillator induced by additive noise. (a) The dependencies of the oscillation variance on the intensity of additive noise b^2 for variables y(labeled 1), y_r (labeled 2), x (labeled 3), and y_{rr} (labeled 4). (b) The dependencies of mean values $\langle y\rangle$ (labeled 1), $\langle y_r\rangle$ (labeled 2), and $\langle y_{rr}\rangle$ (labeled 3) on b^2. *Right:* Additive noise induced phase transition in a nonlinear lattice: predictions of mean field theory. (a) The boundaries of the transition on the plane (σ_ξ^2, D) for different values of a. Clearly seen that by variation of a a point from the dashed region is a point of the transition induced by additive noise. (b) Dependence of order parameter $|\langle x\rangle|$ on additive noise intensity a.

Transitions resulted in the creation of a mean field

Another system where additive noise can induce a transition is a nonlinear lattice of overdamped coupled stochastic oscillators [10,22]. In this system a transition manifests itself in the creation of the mean field and, as a consequence of a special form of coupling, in forming spatially ordered patterns. A system is described by a set of Langevin equations:

$$\dot{x}_{\mathbf{r}} = f(x_{\mathbf{r}}) + g(x_{\mathbf{r}})\xi_{\mathbf{r}} + \mathcal{L}x_{\mathbf{r}} + \zeta_{\mathbf{r}} \quad (7)$$

with f and g defined as

$$f(x) = -x(1+x^2)^2 \quad g(x) = a^2 + x^2 \quad (8)$$

and $\xi_{\mathbf{r}}$, $\zeta_{\mathbf{r}}$ are independent zero-mean-value Gaussian white noises:

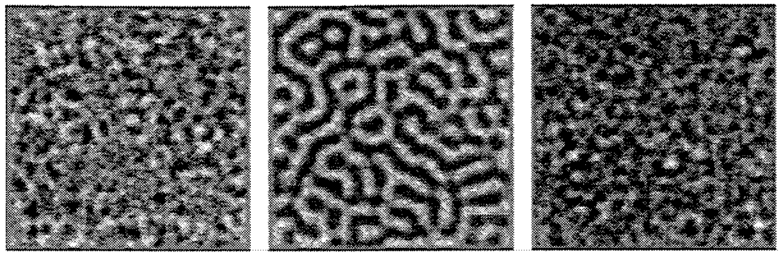

FIGURE 3. Spatial patterns induced by additive noise. From left to the right the intensity of additive noise is increased ($a = 0$). The coordinate of every element in the nonlinear lattice 128x128 is coded from white (minimum) to black (maximum) colours. We find that additive noise causes a reentrant disorder-order-disorder phase transition.

$$\langle \xi_{\mathbf{r}}(t)\xi_{\mathbf{r}'}(t')\rangle = \sigma_\xi^2 \delta_{\mathbf{r},\mathbf{r}'}\delta(t-t') \tag{9}$$

$$\langle \zeta_{\mathbf{r}}(t)\zeta_{\mathbf{r}'}(t')\rangle = \sigma_\zeta^2 \delta_{\mathbf{r},\mathbf{r}'}\delta(t-t') \tag{10}$$

The coupling operator $\mathcal{L}$ is a discretized version of the Swift-Hohenberg coupling term $-D(q_0^2 + \nabla^2)^2$.

To study the influence of the additive noise, we consider two variants: $\zeta_{\mathbf{r}} = 0$ and parameter a is varied, and $a = 0$ and intensity of $\zeta_{\mathbf{r}}$ is varied. These two variants corresponds to different limiting cases of the correlation between additive and multiplicative noise.

Using the approach of the mean field theory, namely substituting the value of the scalar variable $x_{\mathbf{r}'}$ at the sites coupled to $x_{\mathbf{r}}$ by its averaged value $\langle x\rangle$, we get the equation

$$\langle x\rangle = \int x w_{st}(x, \langle x\rangle)dx, \tag{11}$$

where

$$w_{st}(x) = \frac{C(m)}{\sqrt{\sigma_\xi^2 g^2(x) + \sigma_\zeta^2}} \exp\left(2\int_0^x \frac{f(y) - D(y - \langle x\rangle)}{\sigma_\xi^2 g^2(y) + \sigma_\zeta^2}\, dy\right), \tag{12}$$

and $C(\langle x\rangle)$ is the normalization constant.

For details of this calculation see [22]. Solving eq.(11) with parameters D, σ_ξ^2 , and σ_ζ^2, we obtain a boundary between two phases: a disordered ($|\langle x\rangle| = 0$) and ordered one ($|\langle x\rangle| \neq 0$). The boundary of the phase transition on the plane (σ_ξ^2, D) is shown in Fig. 2, *right*, (a), which demonstrates that variation of the intensity of correlated additive noise (parameter a) causes a shift of the transition boundary. The most interesting situation occurs in the dashed region. Here the increase of additive noise intensity causes reentrant disorder-order-disorder phase transition. The corresponding dependence of the order parameter is shown in Fig. 2, *right*,

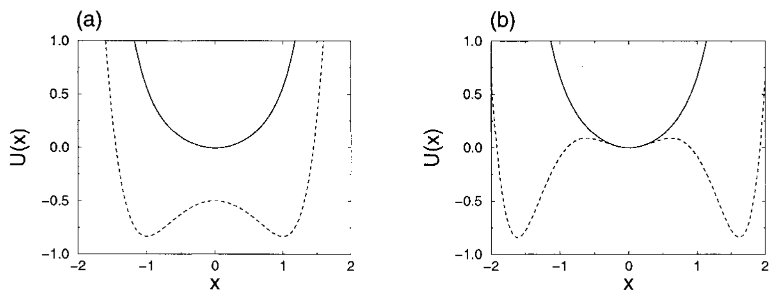

FIGURE 4. Potential for the short-time evolution of the value $\bar{x}$. Two limited cases of correlation of additive noise with multiplicative one: (a) $\zeta = 0$, a parameter a is increased, (b) $a = 0$, an intensity of ζ is increased. The solid line shows potential before a transition, dashed - after. Note that a potential of a system, as well as "stochastic" potential remains monostable.

(b). Due to the special form of coupling, transition to order means in this system a formation of spatially ordered patterns. As shown in [22], qualitatively the same situation is observed for uncorrelated additive noise and $a = 0$. For this case we have shown spatial patterns resulted from the transition (see Fig. 3).

To the discussion about a physical mechanism behind this transition, it is necessary to note that there is no bistability either in the "usual" potential or in the so-called "stochastic" potential. Nevertheless, using some approximations it has been shown [23,22] that the short-time evolution of the average value $\bar{x}$ can be described by the "effective" bistable potential after a transition. A form of this potential before and after a transition is shown in Fig. 4 for two limited cases of correlation between multiplicative and additive noise.

In conclusion, we have shown that a phenomenon, which occurs in the presented model, can be interpreted as *nonequlibrium phase transition induced by additive noise.*

TRANSITIONS INDUCED BOTH BY MULTIPLICATIVE AND ADDITIVE NOISE. STABILIZATION BY ADDITIVE NOISE

In this section we consider a system under the combined action of multiplicative and additive noise. We show that both multiplicative and additive noise can induce a transition, and, what is especially interesting, combination of their actions stabilize noise-induced oscillations. To demonstrate these effects we use a standard epidemiological model for the dynamics of children diseases [24]. Two variants of excitation are possible, either by periodic force [25,26] or by noise [14]. In both

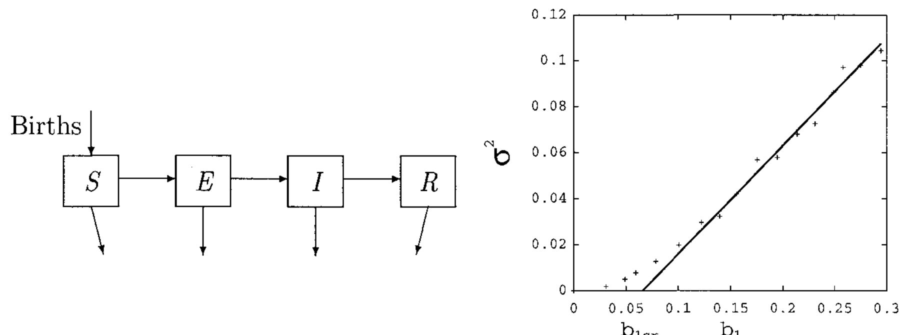

FIGURE 5. *Left:* The scheme of the epidemiological model. *Right:* The dependence of oscillation variance for the variable x on the noise intensity (parameter b_1).

cases the model demonstrates chaotic or noise-induced oscillations which closely resemble oscillations observed in experimental data.

Let us consider a model described in [14] and concentrate our attention on the role of additive noise. The equations of the model are:

$$\dot{S} = m(1-S) - bSI, \quad \dot{E} = bSI - (m+a)E, \quad \dot{I} = aE - (m+g)I \tag{13}$$

where S, E, and I denote the number of susceptible, exposed but not yet infected, and invective children, respectively. The relation between different components is shown in Fig. 5. The parameters $1/m$, $1/a$, $1/g$ are the average expectancy, latency and infection periods of time. The contact rate b is the parameter of excitation and equal to $b = b_0(1 + b_1\xi(t))$ where $\xi(t)$ is a harmonic noise with the peak of spectral density at the frequency 2π and parameter b_1 responsible for the intensity of noise. The excited oscillations will be executed about the stable singular point with the coordinates (S_0, E_0, I_0). Taking it into account one can easily rewrite the equations for new variables $x = S/S_0 - 1$, $y = E/E_0 - 1$, and $z = I/I_0 - 1$ which are deviations from the equilibrium position:

$$\begin{aligned} \dot{x} + mx &= -b_0 I_0 (1 + b_1\xi(t))(x + z + xz) - b_0 b_1 I_0 \xi(t), \\ \dot{y} + (m+a)y &= (m+a)(1 + b_1\xi(t))(x + z + xz) + (m+a) b_1 \xi(t), \\ \dot{z} + (m+g)z &= (m+g)y. \end{aligned} \tag{14}$$

Such form of eqs. clearly shows that the action of noise is multiplicative as well as additive.

An increase of the noise intensity causes noise-induced oscillations of variables S, I, E, which closely resemble experimentally observed data. The oscillations are excited after a noise-induced transition, what can be seen in Fig. 5, *right.* There the variance of oscillations together with an approximating straight line is shown. The point where the straight line crosses the abscissa-axis can be taken as a critical point of the transition. To prove this, we drop artificially the multiplicative noise from eqs.(14). In this case a variance of oscillations is equal to zero if $b_1 < b_{1cr}$ and goes to the infinity shortly after the noise intensity exceeds its critical value. So, additive noise induces a phase transition. The same situation happens if additive noise is absent but multiplicative noise is present.

To conclude, noise-induced transition can be induced both by multiplicative and additive noise in this system. What is even more interesting, the combined action of additive and multiplicative noise performs a stabilization of noise-induced oscillations: in this case the dependence of variance on noise intensity does not go to the infinity.

SUMMARY AND OPEN QUESTIONS

We have reported here recent results concerning the influence of additive noise on noise-induced nonequlibrium phase transitions. We have shown that the role of additive noise can be crucial, that additive noise is able to induce such a transition, strongly influence this transition and stabilize oscillations occured as a result of this transition.

Now let us discuss some open questions and possible directions of future research. The topic of nonequilibrium phase transitions induced by additive noise is rather new. We see three main directions in the study of these transitions:

1. Theory of noise-induced phase transitions. The phenomena described here are demonstrated by a large variety of models, and the question naturally arises whether these transitions belong to any of the existing universality classes. A discussion about it can be found in [23] for the transitions which leads to the breaking of symmetry and creation of the mean field. In general, it is still an open question as well as a question whether dependencies in the presented models are universal for other models demonstrating these transitions. Another interesting problem is a search of combination effects, as, for example, a synthesis of white noise driven ratchets and noise-induced non-equilibrium phase transitions [27], globally synchronized oscillations in subexcitable media [28] or effects in systems with multiplicative, additive noise and external force, which possess properties of stochastic resonance and noise-induced transitions.

2. Experimental confirmation of noise-induced transitions predicted by theory. As proposed in [23], it is worth to reevaluate experiments in physical systems for which noise-induced shifts [3,2] or purely noise-induced transitions may be relevant. Some examples of noise-induced shifts can be mentioned here, such as processes in photosensitive chemical reactions under the influence of fluctuating light intensity [29,30], in liquid crystals [31,32], and in the Raleigh-Benard instability with a fluctuating temperature at the plates [33].

3. Modelling transitions and irregular oscillations observed in experimental data by stochastic models. As shown in [14], already known phenomena which have been explained in the frames of a deterministic theory, could be also successfully described by stochastic models. Note that deterministic and noise-induced processes

are very difficult to be distinguished in many situations. Moreover, sometimes a noisy excitation looks more justified. It is worth to mention a recently outlined hypothesis that turbulence in non-closed flows is a result of noise-induced phase transition [34,35]. Also we expect that noise-induced processes may fit very well modelling of microseismic oscillations [36] or phase transitions observed in physiological systems, especially in bimanual movements [37,38]. Despite the fact that up to now these tempo-induced transitions in the production of polyrhythm are explained by deterministic mechanisms in the presence of noise, we expect that models with noise-induced oscillations will be also relevant in this case. The reason of it is that noise-induced oscillations can be observed in the simplest model of neuron firing [9,18].

Another open question, closely associated with modelling is an identification of the excitation mechanism by the analysis of irregular time-series. This problem is of high importance, because to model a system one should know the physical mechanism of an excitation. At the same time, time-series are often the single source of the information about a nonlinear system - "black box". At this point, it is essential to note that classical methods of analysis, such as a spectral analysis or a calculation of a correlation dimension are sometimes unable to distinguish between noise-induced irregular oscillations and chaotic oscillations of the deterministic nature [12].

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