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On–Off Intemittency Phenomena in a Pendulum with a Randomly Vibrating Suspension Axis

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Abstract—We demonstrate that the excitation of noise-induced oscillations of a pendulum with a randomly vibrating suspension axis occurs via 'on–off intermittency'. The dependence of the mean duration of 'laminar' phase on the excess of the noise intensity over a certain critical value is calculated analytically. These theoretical dependencies are confirmed by the results of numerical experiments. Further, we study the influence of an additional harmonic action. It is shown that the oscillations are intensified if the frequency of this action is low, and suppressed if the action frequency is high. It is interesting that the suppression of the oscillations, much like their excitation, occurs via 'on–off intermittency'. The dependence of the mean duration of 'laminar' phase on the action amplitude is obtained from numerical experiments.

1. INTRODUCTION

Noise-induced phase transitions in a pendulum with a randomly vibrating suspension axis have been recently considered in detail in [1, 2]. This transition consists in the excitation of pendulum's oscillations and the birth of an induced attractor owing to the random vibration of the suspension axis. It has been detected numerically that the excitation of the pendulum's oscillations occurs via intermittency of a peculiar kind. In this contribution we study this kind of intermittency more thoroughly. It is shown that it is the so-called on–off intermittency and it statistical characteristics are calculated, both analytically and numerically in Section 2.

Next, the influence of an additional harmonic action is studied. This problem is very important for controlling noise-induced oscillations. We demonstrate that the low-frequency action intensifies the noise-induced pendulum's oscillations, whereas the high-frequency action suppresses them. In a certain region of the frequencies, the synchronization of the pendulum's oscillations by additional action takes place; the synchronization here is understood as a coincidence of the mean frequency of noise-induced oscillations and that of external force. The characteristics of this phenomenon are studied in Section 3. In Section 4 we conclude our results.

2. EXCITATION OF THE OSCILLATIONS OF A PENDULUM OWING TO RANDOM VIBRATION OF ITS SUSPENSION AXIS. ON–OFF-INTERMITTENCY

In [1, 2] we have studied, both analytically and numerically, the equation for oscillations of a pendulum with a randomly vibrating suspension axis. Now we generalize this system by taking into account additive noise. The equation then reads as

$$\ddot{\varphi} + 2\beta(1 + \alpha\dot{\varphi}^2)\dot{\varphi} + \omega_0^2(1 + \xi_1(t)) \sin \varphi = k\xi_2(t), \tag{1}$$

where φ is the pendulum's angular deviation from the equilibrium position, $2\beta(1 + \alpha\dot{\varphi}^2)\dot{\varphi}$ is the value proportional to the torque of the friction force which is assumed to be nonlinear, ω_0 is the natural frequency of small pendulum's oscillations, $\xi_1(t)$ is the acceleration of the suspension axis that is a comparatively wide-band random process with nonzero power spectrum density at the frequency $2\omega_0$, and $k\xi_2(t)$ is the additive white noise.

First of all let us consider the case when the additive noise is absent, i.e. $k=0$. Setting $\varphi \approx A \cos(\omega_0 t + \phi)$ and solving eqn (1) approximately by the Krylov–Bogolyubov method, we obtain the following truncated equations for the amplitude A and phase ϕ of the oscillations [1, 2]†

$$\dot{A} = \frac{\omega_0^2 K_1}{4}\left(\eta - \frac{3\beta\tilde{\gamma}}{\omega_0^2 K_1}A^2\right)A + \frac{\omega_0}{2}A\zeta_1(t), \tag{2}$$

$$\dot{\phi} = \omega_0 M - \frac{3}{8}\omega_0\tilde{\gamma}A^2 + \omega_0\zeta_2(t), \tag{3}$$

where

$$\eta = 1 - \frac{4\beta}{\omega_0^2 K_1}$$

is the parameter characterizing the extent to which the noise intensity is in excess of its critical value for which the phase transition arises, $\tilde{\gamma} = \gamma + \alpha\omega_0^2$, γ is the coefficient of the first nonlinear term in the expansion of $\sin \varphi$, $K_1 = \kappa(2\omega_0)/2$, is the parameter characterizing the intensity of suspension axis' vibration, $\kappa(2\omega_0)$ is power spectrum density of the noise $\xi_1(t)$ at the frequency $2\omega_0$, $\zeta_1(t)$ is a random process with zero mean value and the intensity K_1, $M = \langle\overline{\xi_1 \cos^2\psi}\rangle$, and $\zeta_2(t)$, much like to $\zeta_1(t)$, is a random process with zero mean value and the intensity $K_2 = (\kappa(0) + K_1)/4$. The value of M depends on the characteristics of the random process $\xi_1(t)$: if $\xi_1(t)$ is a white noise then $M=0$, but if $\xi_1(t)$ has a finite correlation time, e.g. its power spectrum density is

$$\kappa(\omega) = \frac{\delta^2\kappa(2\omega_0)}{(\omega - 2\omega_0)^2 + \delta^2},$$

then

$$M = -\frac{\delta\omega_0^2\kappa(2\omega_0)}{4(16\omega_0^2 + \delta^2)}.$$

It is seen from this expression and from eqn (3) that in the last case the mean frequency of oscillations is shifted to low-frequency region with the increase of noise intensity. This fact is supported by the results of numerical experiment.

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†These equations are valid only for sufficiently small η.

The Fokker–Planck equation associated with eqns (2) and (3) is

$$\frac{\partial w(A,\phi)}{\partial t} = -\frac{\partial}{\partial A}\left(\left(\frac{\omega_0^2 K_1}{4}\eta - \frac{3}{4}\beta\tilde{\gamma}A^2\right)Aw(A,\phi)\right) - \omega_0\left(\frac{3}{8}\gamma A^2 - M\right)\frac{\partial w(A,\phi)}{\partial \phi}$$

$$+\frac{K_1\omega_0^2}{8}\frac{\partial^2}{\partial A^2}(A^2 w(A,\phi)) + \frac{K_2\omega_0^2}{2}\frac{\partial^2 w(A,\phi)}{\partial \phi^2}. \quad (4)$$

Finding from this equation the steady-state probability density for the amplitude A, we can calculate the mean value of the steady-state amplitude and show that for $\eta > 0$ the parametric excitation of pendulum's oscillations occur under the influence of noise. This manifests itself in the fact that the mean value of the amplitude and the variance of the pendulum's angular deviation become different from zero [1, 2]. This phenomenon is interpreted as a noise-induced phase transition and the birth of an induced attractor. The latter follows, firstly, from the form of the truncated equation for the oscillation amplitude (see eqn (2)) which is similar to the truncated equation for the oscillation amplitude of a noisy van der Pol generator, and, secondly, from the fact that the correlation dimension of the attractor constructed from the data obtained by the numerical simulation by using the Takens' technique turns out to be finite [1, 2]. However, this attractor is very noisy. This is made evident by the fact that the embedding dimension calculated by both well-adapted basis technique [3] and Broomhead–King's method [4] is rather large.

Numerical simulation of eqn (1) showed that, as one would expect from the theoretical results, when the intensity of the suspension axis vibration, characterized by the value of $\kappa(2\omega_0)$, exceeds a certain critical value κ_{cr} proportional to the friction factor β, the excitation of pendulum's oscillations occurs. Examples of such oscillations found by numerical simulation of eqn (1) are depicted in Fig. 1. It is seen from this figure that close to the excitation threshold the pendulum's oscillations possess the property of a peculiar kind of intermittency†, i.e. over prolonged periods the pendulum oscillates in the immediate vicinity of its equilibrium position (so-called 'laminar' phases); these slight oscillations alternate with short strong bursts ('turbulent' phases). Away from the threshold the duration of laminar phases decreases and that of turbulent ones increases, and finally laminar phases disappear. The variance of the pendulum's angular deviation increases during this process.

Interestingly enough that in the case of the pendulum, the intermittency observed is different from all types of the intermittency described in [5–7], although they are similar in their external manifestations. It is so-called 'on–off intermittency'. This term was recently introduced by Platt *et al.* in [8], though a map associated with the similar type of intermittent behavior was first considered by Pikovsky [9] and then by Fujisaka and Yamada [10]. It is essential that this type of intermittency can occur not only in dynamical systems but in stochastic systems as well [11]. In [11] the statistical properties of on–off intermittency were obtained from the analysis of the map

$$x_{n+1} = a(1 + z_n)x_n + f(x_n),$$

where z_n is either a certain deterministic chaotic process or a random process, a is the bifurcation parameter, and $f(x_n)$ is a nonlinear function. For this map it was shown that the mean duration of laminar phases has to be proportional to a^{-1}.

Let us calculate the mean duration of laminar phase in our system, i.e. using eqns (2) and (3) and the Fokker–Planck eqn (4) associated with them. We assume that the pendulum oscillates in a laminar phase if the oscillation amplitude A is not larger than a certain value ϵ. Then the mean duration of the laminar phase τ_ϵ is determined by the mean duration of a

†The intermittency phenomenon is described, for example, in [5–7].

random walk-like motion of a representative point inside the circle of radius ϵ on the plane φ, $\dot{\varphi}$. This duration can be calculated (see [12, 13,6]) using the steady-state solution of eqn (4) with the boundary condition

$$w(A,\phi)\,|_{A=\epsilon}=0. \tag{5}$$

Because the value of ϵ is assumed to be small, we can neglect the term $(3/4)\beta\tilde{\gamma}A^2$ in eqn (4). In so doing the solution of eqn (4) with the boundary condition (5) is

$$w(A,\phi)=\frac{8G_0}{\omega_0^2K_1(1-2\eta)A}(\epsilon^{1-2\eta}A^{2\eta-1}-1), \tag{6}$$

where G_0 is the value of the probability flow

$$G=\frac{\omega_0^2K_1}{4}\left(\eta Aw-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}A}(A^2w)\right)$$

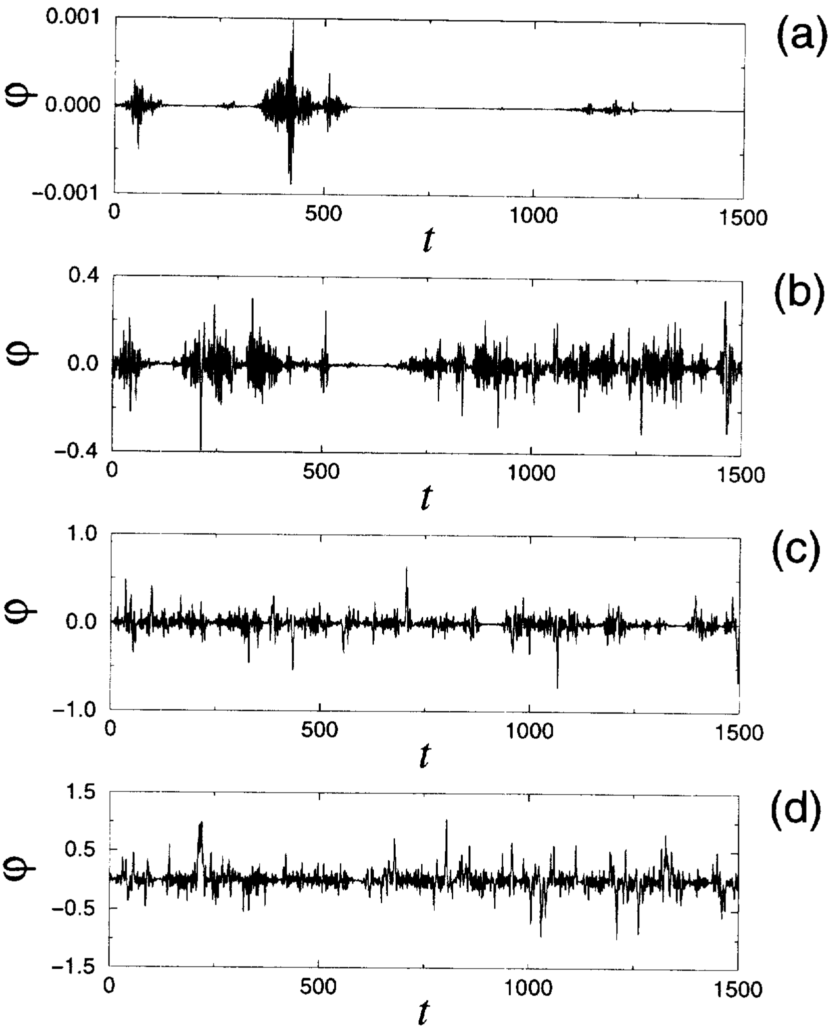

Fig. 1. The time dependence of the pendulum angle $\varphi(t)$ for $\omega_0=1$, $\beta=0.1$, $\alpha=100$, $(k=0)$, and different values of noise intensity: (a) $\kappa(2)/\kappa_{cr}=1.01$, (b) $\kappa(2)/\kappa_{cr}=1.5$, (c) $\kappa(2)/\kappa_{cr}=2.44$, and (d) $\kappa(2)/\kappa_{cr}=6.25$.

across any circumference inside the circle of radius ϵ. The value of G_0 is determined from the normalization condition by integrating the expression (6) over the circle of radius ϵ. It was shown in [12, 13] that the mean duration of reaching the circle boundary is equal to G_0^{-1}. Taking into account that the representative point touching the boundary of the circle can return back with a certain probability p, we obtain for the mean duration of the laminar phase τ_ϵ the following expression:

$$\tau_\epsilon = G_0^{-1}(1-p)\sum_{j=1}^{\infty} jp^{j-1}.$$

Summarizing the series and having regard to the expression for G_0, we get

$$\tau_\epsilon = G_0^{-1}(1-p)^{-1} = \frac{8\pi\epsilon}{\omega_0^2 K_1 \eta(1-p)} = \frac{16\pi\epsilon}{\omega_0^2(\kappa(2\omega_0)-\kappa_{cr})(1-p)}. \tag{7}$$

It is evident that $p < 1/2$ because the force which acts upon the representative point on the circle boundary is directed outside the circle. For small η and ϵ this force is very small and p is close to 1/2. In this case $\tau_\epsilon = 2G_0^{-1}$. As η and ϵ increase the value of p decreases.

It follows from (7) that for small η and ϵ the mean duration of the laminar phase has to be proportional to ϵ and inversely proportional to η. This result agrees quite well with the findings for the rather simple map of [11]. Numerical simulation of eqn (1) showed that for small η and ϵ the formula (7) with $p = 1/2$ is valid in a good approximation; whereas for larger ϵ some discrepancies are observed.

The dependencies of τ_ϵ on $\kappa(2\omega_0) - \kappa_{cr}$ for two values of ϵ calculated by numerical simulation are given in Fig. 2. The corresponding theoretical dependencies are shown by solid lines. A discrepancy between theoretical and numerical dependencies for large value of $\kappa(2\omega_0) - \kappa_{cr}$ is caused by the nonlinear dependence of p on $\kappa(2\omega_0) - \kappa_{cr}$ and by the fact that theoretical calculations are valid for small ϵ only.

The excitation of oscillations of a pendulum owing to random vibration of its suspension axis makes itself evident in the fact that the variance of the pendulum's angular deviation becomes nonzero. The dependence of the variance of φ on the relative power spectrum density $\kappa(2\omega_0)/\kappa_{cr}$ is shown in Fig. 3 both in the absence of additive noise (squares) and in its

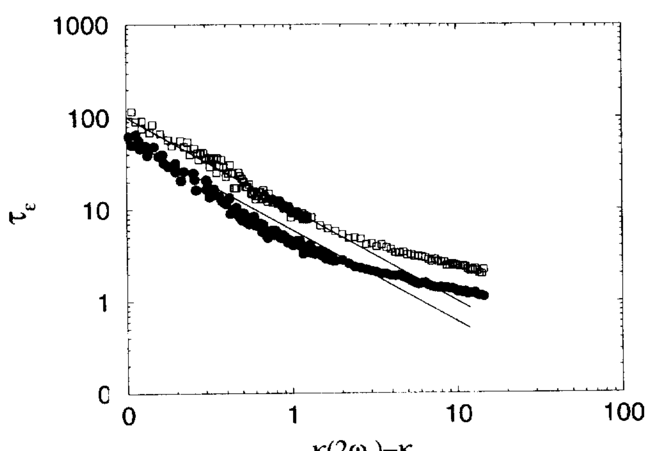

Fig. 2. The dependencies of the mean duration of the laminar phase τ_ϵ on the excess of the noise intensity over its critical value $\kappa(2\omega_0) - \kappa_{cr}$ for $\epsilon = 0.06$ (full circles) and $\epsilon = 0.1$ (squares) from numerical simulation. The corresponding theoretical dependencies are shown by solid lines.

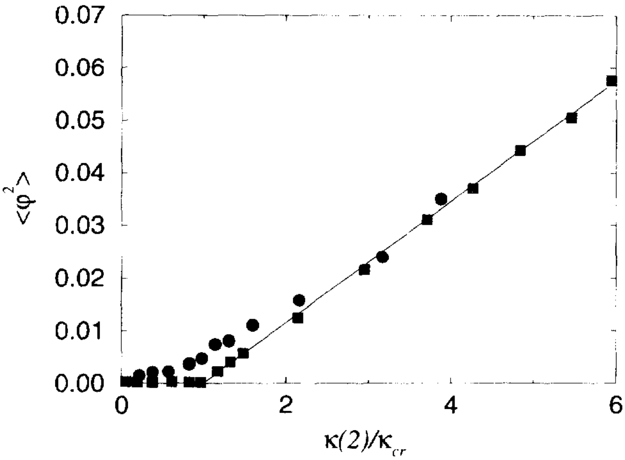

Fig. 3. The dependence of $\overline{\varphi^2}$ on $\kappa(2)/\kappa_{cr}$ in the absence of additive noise (squares) and with additive noise for $\overline{k^2\xi_2^2} = 0.0025\ \overline{\xi_1^2}$ (circles) (the straight line $\overline{\varphi^2} = 0.01151\ (\kappa(2)/\kappa_{cr}(2) - 1)$ is shown as a solid line).

presence (circles). We see that in the presence of additive noise the excitation threshold gets fuzzy, and the dependence of $\overline{\varphi^2}$ on $\kappa(2\omega_0)/\kappa_{cr}$ becomes smooth. In this case the on–off intermittency can be observed only when the intensity of multiplicative noise is under its threshold value κ_{cr} (see Fig. 4).

3. THE INFLUENCE OF AN ADDITIONAL HARMONIC ACTION ON THE NOISE-INDUCED PENDULUM'S OSCILLATIONS

Numerical simulation of eqn (1) with an additional harmonic action shows that this action intensifies the pendulum's oscillations in the case when the action frequency is low. Moreover, if the intensity of the suspension axis random vibration is under its threshold value, then a small additional low-frequency action initiates the excitation of noise-induced pendulum's oscillations.

In the case when the frequency of an additional harmonic action is sufficiently high, the suppression of noise-induced pendulum's oscillations occurs rather than their intensification. The results of numerical simulation of eqn (1) with $\xi_1 + a\cos\omega_a t$ in place of ξ_1, where $\omega_a >> 1$, are represented in Fig. 5 for the case when additive noise is absent ($k = 0$). We see that, for small amplitudes of the high-frequency action, this action has only a small or no effect on the noise-induced oscillations. As the action amplitude increases the intensity of the noise-induced oscillations decreases rapidly and for a certain value of the action amplitude

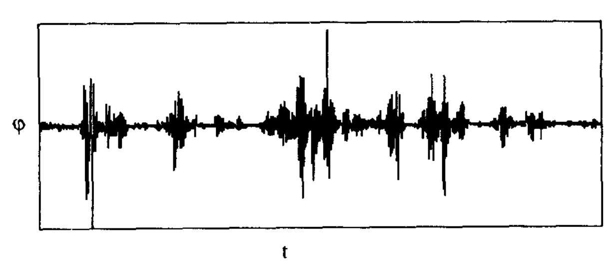

Fig. 4. On–off intermittency in the presence of the additive noise. The intensity of the multiplicative noise is below the threshold of exitation of noise-induced oscillations: the dynamics of $\varphi(t)$ is shown for $\overline{k^2\xi_2^2} = 0.000125\ \overline{\xi_1^2}$, $\kappa(2)/\kappa_{cr} = 0.92$.

the oscillations are suppressed entirely. It is important to note that the suppression of the oscillations, like their excitation, occur via 'on–off intermittency'. As the action amplitude increases, the duration of 'laminar' phases increases too. The dependence of the mean duration of 'laminar' phases τ on the multiplicative action amplitude a found numerically for $\epsilon = 0.002$, $\kappa(2\omega_0)/\kappa_{cr} = 6.25$ is shown in Fig. 6. Solid line shows the dependence $\tau = 400\epsilon/(\kappa(2\omega_0) - \kappa_{cr} - 0.00056a^2)$; such a character of the dependence follows from theoretical considerations. We see that this dependence fits the experimental data rather well.

We emphasize that the suppression of a noise-induced pendulum's oscillations occurs not only due to a multiplicative harmonic action, but due to an additive one as well. In the last case, the action is found to be more effective. For example, the decrease of the oscillation variance from 0.18 to 0.13 due to a high-frequency action can be achieved if the amplitude of the additive force is equal to 1. For the same suppression effect the amplitude of the multiplicative force should be 8 times larger.

It is evident that, if additive noise is present, then the noise-induced pendulum's oscillations cannot be entirely suppressed, but the suppression can be impressive. This is seen from Fig. 7 depicting the process $\varphi(t)$ and $\dot{\varphi}(t)$ for two values of the amplitude of action at the frequency 19.757 (cf. with Fig. 5).

Next, we check whether this additive action leads to synchronization of the noise-induced oscillations. The possibility of synchronization and its character can be demonstrated

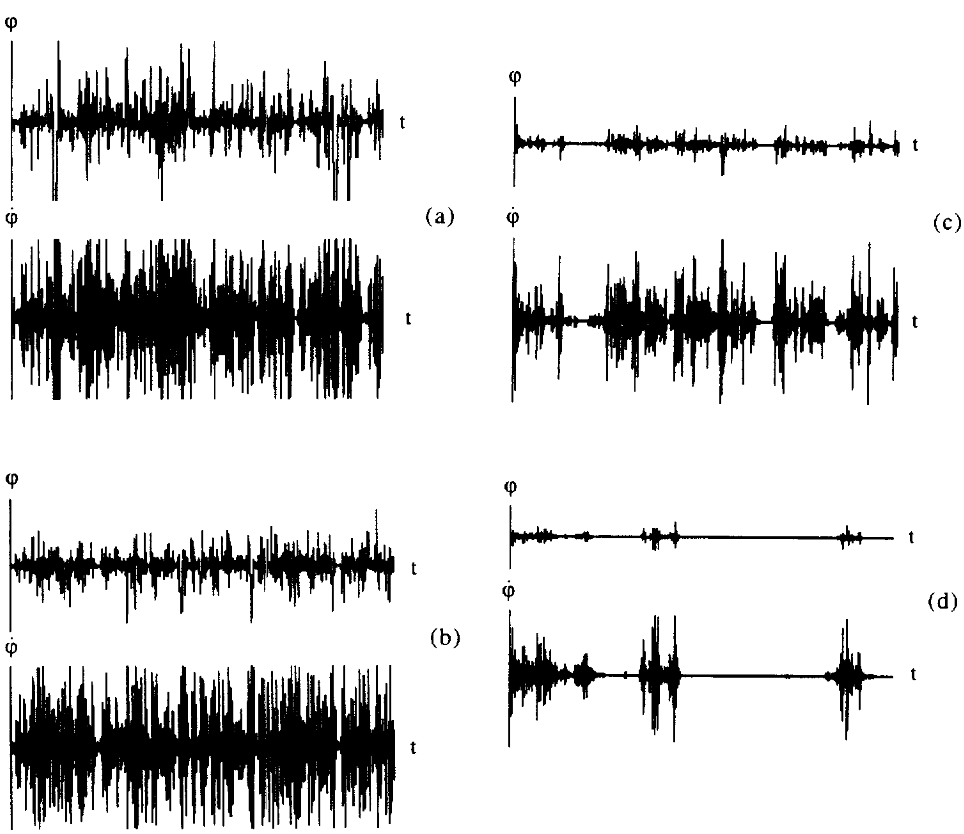

Fig. 5. The dependencies of $\varphi(t)$ and $\dot{\varphi}(t)$ on different amplitudes a of the additional harmonic action: (a) $a=5$, (b) $a=15$, (c) $a=30$, and (d) $a=40$, ($\omega_0=1$, $\beta=0.1$, $\alpha=100$, $\kappa(2)/\kappa_{cr}(2)=5.6$, $k=0$, $\omega_a=19.757$). The increase of the amplitude a leads to the suppression of pendulum's oscillations.

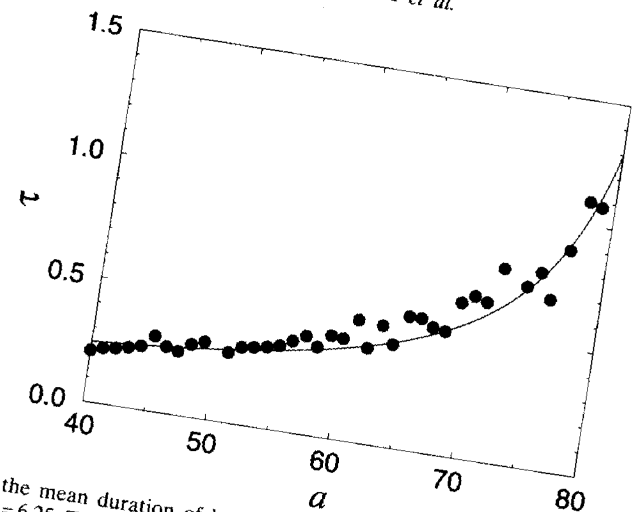

Fig. 6. The dependence of the mean duration of laminar a phases τ on the multiplicative action amplitude a for $\epsilon = 0.002$, $\kappa(2\omega_0)/\kappa_{cr} = 6.25$. The solid line shows the dependence $\tau = 400\epsilon/(\kappa(2\omega_0) - \kappa_{cr} - 0.00056a^2)$.

analytically. In the absence of additive noise but with an additive harmonic action, eqn (1) becomes

$$\ddot{\varphi} + 2\beta(1 + \alpha\dot{\varphi}^2)\dot{\varphi} + \omega_0^2(1 + \xi_1(t)) \sin\varphi = \omega_0^2 B \cos\omega t. \tag{8}$$

Setting $\varphi \approx A\cos(\omega t + \phi)$ and solving eqn (8) approximately by the Krylov–Bogolyubov method we obtain the following truncated equations for the amplitude A and phase ϕ:

$$\dot{A} = \frac{\omega^2 K_{1\omega}}{4}\left(\eta_\omega - \frac{3\beta\tilde{\gamma}}{\omega^2 K_{1\omega}}A^2\right)A - \frac{\omega B}{2}\sin\phi + \frac{\omega}{2}A\zeta_1(t), \tag{9}$$

$$\dot{\phi} = \Delta_0 + \omega M_\omega - \frac{3}{8}\omega\tilde{\gamma}A^2 - \frac{\omega B}{2A}\cos\phi + \omega\zeta_2(t), \tag{10}$$

where $\eta_\omega = 1 - 4\beta/\omega^2 K_{1\omega}$, $\tilde{\gamma} = \gamma + \alpha\omega^2$, $K_{1\omega} = \kappa(2\omega)/2$, $\zeta_1(t)$ is a random process with zero mean value and the intensity $K_{1\omega}$, $\Delta_0 = (\omega_0^2 - \omega^2)/2\omega$ is the frequency mistuning, $M_\omega = \langle\xi_1\cos^2(\omega t + \phi)\rangle$, and $\zeta_2(t)$, much like to $\zeta_1(t)$, is a random process with zero mean value and the intensity $K_{2\omega} = (\kappa(0) + K_{1\omega})/4$. The value of M_ω can be calculated in a similar way as for eqn (3).

In the case when the action amplitude is sufficiently small we can assume that the oscillation amplitude reaches a steady state value considerably faster than the phase and its steady state value is close to that as the harmonic action is absent. Then eqn (10) can be rewritten in the form

$$\dot{\phi} = \Delta(A_0^2) - \Delta_s(A_0)\cos\phi + \omega\zeta_2(t), \tag{11}$$

where $\Delta(A_0^2) = \Delta_0 + \omega M_\omega - (3/8)\omega\tilde{\gamma}A_0^2$ is the effective frequency mistuning depending on the steady state value of amplitude A_0 in the absence of an additional action, $\Delta_s(A_0) = \omega B/2A_0$ is the effective half-width of the synchronization region which also depends on A_0.

Equation (11) coincides in its form with eqn (11.3.1) of the book [13] derived for the synchronization problem in a van der Pol generator with an external harmonic force and additive noise. The only distinction is that $\Delta(A_0^2)$ and $\Delta_s(A_0)$ depend on A_0. Therefore, the results obtained in [13] must be averaged over the amplitude A_0 with using the steady state probability distribution found in [1, 2]. As a result, the region of the frequency mi... within which the mean frequency of the pendulum's oscill...

frequency, can be found. It should be noted that in the synchronous regime the oscillations of the pendulum remain irregular, i.e. only the frequency of these oscillations is approximately entrained by the external action, whereas their amplitude remains random. This is seen in Fig. 8, where an example of the time dependencies of φ and the external force is given. Comparing Fig. 8(a) with Fig. 1(c) we can conclude that the external force intensifies the oscillations and makes them more ordered.

Such an approach was used in [14–16], where the effects of phase and frequency locking in chaotic systems have been studied numerically. Here we demonstrate that synchronization of such a kind can be observed in systems with noise-induced oscillations as well.

The mean frequency is defined as $\Omega = \langle \dot{\psi} \rangle = \omega + \langle \dot{\phi} \rangle$, where the instantaneous phase ψ is determined by means of the analytical signal approach based on the Hilbert transform (the description of the technique and references can be found in [14,16]). In Fig. 9(a) we plot the differences between the mean frequency of pendulum oscillations Ω and the frequency of external force ω vs ω for different values of the force amplitude B. From this plot one can see, that if B is large enough, then $\Omega \approx \omega$ in some range of ω, i.e. frequency entrainment occurs. These dependencies are similar to those known for synchronization of the van der Pol

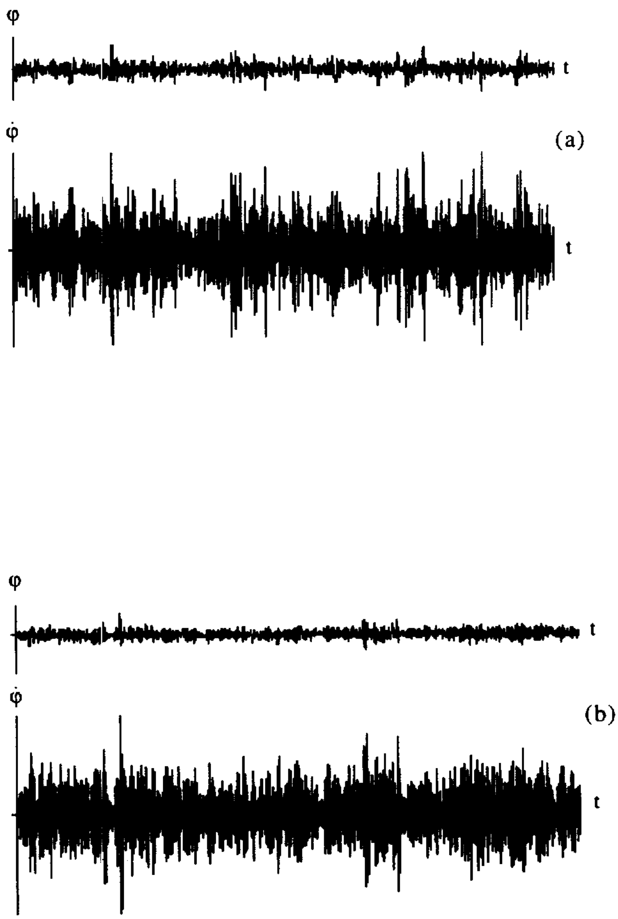

Fig. 7. The dependencies of $\varphi(t)$ and $\dot{\varphi}(t)$ for $\overline{k^2\xi_2^2} = 0.05\overline{\xi_1^2}$. (a) $a = 40$ and (b) $a = 50$; the remaining parameters are the same as in Fig. 5. It is seen that in the presence of the additive noise pendulum's oscillations can not been entirely suppressed.

generator in the presence of noise [17, 18,13]. We consider the system to be synchronized, if $|\Omega - \omega| < \varepsilon$, where ε is a certain threshold value. In this way we obtain synchronization regions in the parameter plane (ω, B) (Fig. 10) for different values of the noise intensity. We see that, as the noise intensity increases the synchronization regions are shifted to lower frequency regions.

We note, that in contrast to synchronization of chaotic systems [14,16], frequency locking of the pendulum's oscillations occur for rather strong driving only (amplitude of the external force is of the same order or larger than the mean amplitude of the pendulum's oscillations in the absence of the harmonic action).

It is very interesting that, as differentiated from the ordinary synchronization of periodic self-oscillatory systems, the intensity of oscillations at the center of synchronization region has no maximum but decreases monotonically as the external force frequency increases (see Fig. 9(b)).

4. CONCLUSIONS

To summarize, we have shown that the excitation of noise-induced oscillations of a pendulum with a vibrating suspension axis is accompanied by on–off intermittency. The statistical properties of this intermittency have been obtained by solving the first passage

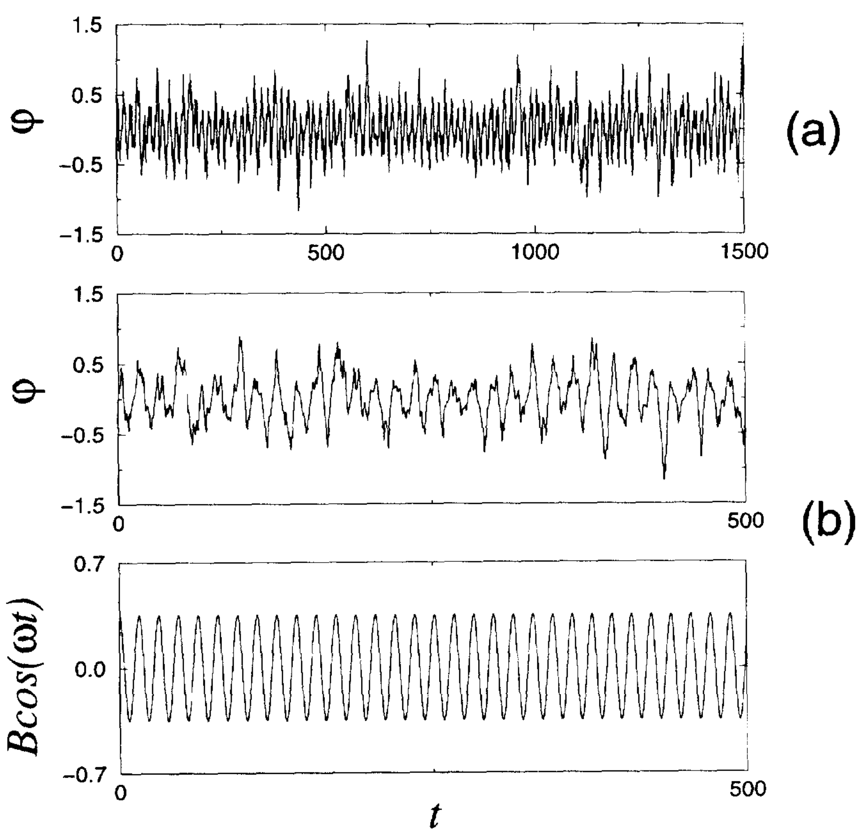

Fig. 8. (a) The time dependence of φ in a center of the synchronization region ($\omega = 0.4$) for $\kappa(2)/\kappa_{cr} = 2.44$, $B = 0.35$. The enlarged part of this dependence is shown in (b) along with the external force. The plots illustrate the approximate equality of the frequencies of the pendulum's oscillations and of the external force. Note that synchronized oscillations remain irregular.

problem using the Fokker–Planck equation. These results are supported by numerical simulation.

Further, we have studied the behavior of the pendulum under periodic external action, both multiplicative and additive. We have found two effects: low-frequency action results in intensification, or even initiation, of noise-induced pendulum's oscillation, whereas the high-frequency action suppresses them. These phenomena can be considered as analogues of classical effects of asynchronous excitation and quenching. A very interesting finding is that the suppression, much like the excitation, occurs via on–off intermittency.

In a certain range of the action frequencies the synchronization of noise-induced oscillation takes place. This phenomenon was justified by the analysis of truncated equations for amplitude and phase of oscillations.

The analysis of on–off intermittency is very important for understanding of the onset of turbulence. The analogy of intermittency of such a kind, initially found for a two-dimensional

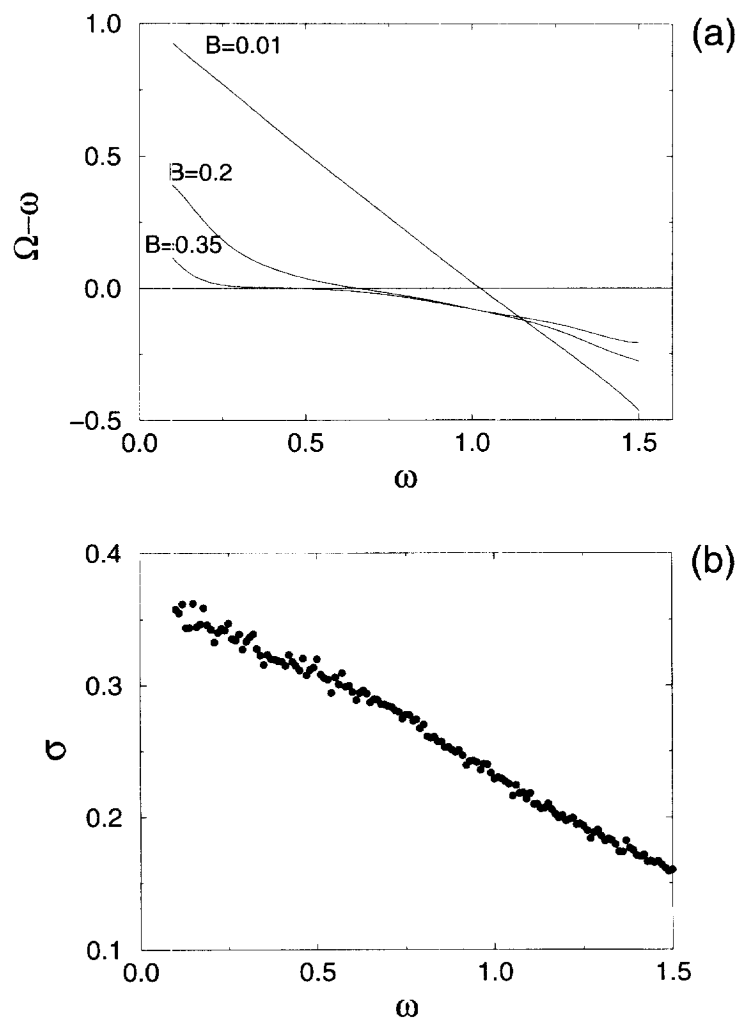

Fig. 9. (a) The differences between the mean frequency of the pendulum's oscillations Ω and the external force frequency ω vs ω for different values of the force amplitude B (the values of B are indicated close to the corresponding curve. The region of synchronization, where $\Omega \approx \omega$, is clearly seen for sufficiently strong external action (b). The plot of $\sigma = \overline{\varphi^2}^{1/2}$ vs ω for $B = 0.35$.

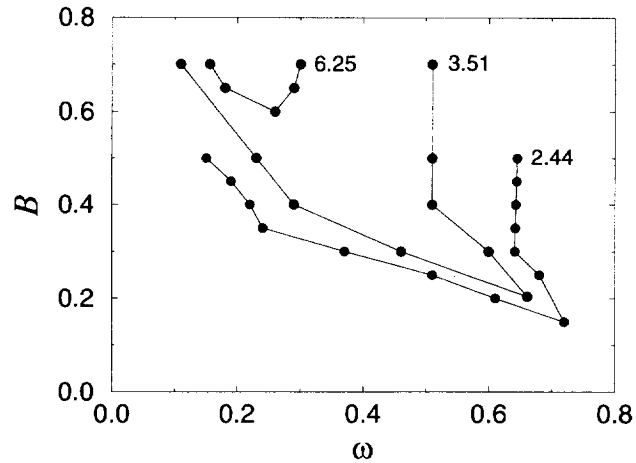

Fig. 10. The synchronization regions for different values of excess of the noise intensity over the critical value (the values of $\kappa(2)/\kappa_{cr}$ are indicated near the corresponding curves). With the increase of the noise intensity the synchronization regions are shifted to a lower frequency region.

map, with turbulent intermittency was noted by Fujisaka and Yamada [10] and later discussed in [19, 20]. The parallels between the control of pendulum's oscillation and suppression or intensification of turbulent processes in subsonic jets are reported elsewhere [20].

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REFERENCES

1. Landa, P. S. and Zaikin, A. A., Noise-induced phase transitions in a pendulum with a randomly vibrating suspension axis. *Phys. Rev. E,* 1996, **54,** 3535–3544.
2. Landa, P.S. and Zaikin, A.A., Nonequilibrium noise-induced phase transitions in simple systems, *Zh. Exp. Teoret. Fiz.,* 1997 **84**(1), 197–208 .
3. Landa, P. S. and Rosenblum, M. G., Time series analysis for system identification and diagnostics. *Physica D,* 1991, **48,** 232–254.
4. Broomhead, D. S. and King, G. P., Extracting qualitative dynamics from experimental data. *Physica D,* 1986, **20,** 217–236.
5. Schuster, H. G., *Deterministic Chaos.* Physica, Weinheim, 1984.
6. Neimark, Yu. I., and Landa, P. S., *Stochastic and Chaotic Oscillations,* Nauka, Moscow, 1987 (in Russian); English translation: Kluwer, Dordrecht, 1992.
7. Manneville, P., *Dissipative Structures and Weak Turbulence.* Academic Press, New York, 1990.
8. Platt, N., Spiegel, E. A. and Tresser, C., On-off intermittency: a mechanism for bursting. *Phys. Rev. Lett.* 1993, **70,** 279–282.
9. Pikovsky, A. S., On the interaction of strange attractors. *Z. Phys. B, Condenced Matter,* 1984, **55,** 149–154.
10. Fujisaka, H. and Yamada, T. A new intermittency in coupled dynamical systems. *Progr. Theor. Phys.,* 1985, **74,** 918–921.
11. Heagy, J. F., Platt, N. and Hammel, S. M., Characterization of on-off intermittency. *Phys. Rev., E,* 1994, **49,** 1140–1150.
12. Landa, P. S. and Stratonovich, R. L., The theory of fluctuational transitions of different systems from one steady state to another. *Vestnik MGU,* 1962, **1,** 33–45 (in Russian)
13. Landa, P. S., *Self-Oscillations in Systems with Finite Number of Degrees of Freedom.* Nauka, Moscow, 1980 (in Russian).
14. Rosenblum, M. G., Pikovsky, A. S. and Kurths, J., Phase synchronization of chaotic oscillators. *Phys. Rev. Lett.,* 1996, **76,** 1804–1807.
15. Pikovsky, A. S., Rosenblum, M. G. and Kurths, J., Synchronization in a population of globally coupled chaotic oscillators. *Europhys. Lett.,* 1996, **34,** 165–170.
16. Pikovsky, A. S., Rosenblum, M. G., Osipov, G. V. and Kurths, J., Phase synchronization of chaotic oscillators by external driving. *Physica D,* 1997, **104**(3–4), 219–238.
17. Stratonovich, R. L., *Selected Problems of the Fluctuations Theory in Radioengineering.* Sovetskoe Radio,

Moscow, 1961 (in Russian).
18. Stratonovich, R. L., *Topics in the Theory of Random Noise*. Gordon and Breach, New York, 1963.
19. Landa, P. S., Turbulence in nonclosed fluid flows as a noise-induced phase transition. *Europhys. Lett.*, 1996, **36**(6), 401–406.
20. Landa, P. S., Ginevsky, A. S., Vlasov, Ye. V. and Zaikin, A. A., Turbulence and coherent structures in subsonic submerged jets. Accepted for publication in *Int. J. Bifurc. and Chaos*.