Robust uncertainty quantification of tsunami ionospheric holes for the 2011 Tohoku-Oki earthquake using satellite data.

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Space-based system using GPS satellites could warn of incoming tsunamis

28 April 2022

A new method for detecting tsunamis using existing GPS satellites orbiting Earth could serve as an effective warning system for countries worldwide, according to a new study by an international team led by UCL researchers.
SPACE-BASED SYSTEM USING GPS SATELLITES COULD WARN OF INCOMING TSUNAMIS

Source(s): University College London

28 April 2022
GPS signals could detect tsunamis better and faster than seismic sensors

Stefanie Waldek
May 5, 2022 · 3 min read

GPS networks are already a crucial part of everyday life around the world, but an international team of scientists has found a new, potentially life-saving use for them: tsunami warnings.
Collaboration

Space Science
UCL's Mullard Space Science Laboratory

Statistics
UCL’s Statistical Science department

Seismology
Tokai University

Electrodynamics
University of Shizuoka

Space-based analysis
11th March 2011

[Time] 05:46:24 (UTC)

[Location] 38.297°N 142.373°E

[Depth] 29.0 km

[Magnitude] 9.1
Background

11th March 2011

<table>
<thead>
<tr>
<th>Location</th>
<th>3 minutes after</th>
<th>28 minutes after</th>
<th>44 minutes after</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aomori</td>
<td>1 meter</td>
<td>3 meters</td>
<td>8 meters</td>
</tr>
<tr>
<td>Iwate</td>
<td>3 meters</td>
<td>6 meters</td>
<td>More than 10 meters</td>
</tr>
<tr>
<td>Miyagi</td>
<td>6 meters</td>
<td>More than 10 meters</td>
<td>More than 10 meters</td>
</tr>
<tr>
<td>Fukushima</td>
<td>3 meters</td>
<td>6 meters</td>
<td>More than 10 meters</td>
</tr>
</tbody>
</table>

Tsunami warning
Background

How to overcome?

Traditional Seismology

- Magnitude cannot be estimated.

Distribute buoys densely in the ocean

- Impossible due to budget constraints. No ship can sail.

Distribute instruments on the seafloor

- Impossible to know where earthquakes occur in advance.
A possible solution

Total Electron Content (TEC)

Ionospheric thin layer

300km

GPS receiver

Land

Sea

GPS satellite
A possible solution

- **Total Electron Content (TEC)**
- **Ionospheric thin layer**
- **300km**
- **GPS satellite**
- **GPS receiver**
- **Earthquake**
A possible solution

Ionospheric thin layer

300km

GPS receiver

GPS satellite

Initial tsunami

Earthquake

Sea
A possible solution

Ionospheric thin layer

300km

GPS receiver

Initial tsunami

Earthquake

Sea

GPS satellite
A possible solution

Ionospheric thin layer

300km

GPS receiver

TEC decreases

Initial tsunami

Earthquake

Sea
A possible solution

Initial tsunami

Earthquake

Sea

 gps satellite

Ionospheric thin layer

recombination

- lor receiver

Background

GPS
A possible solution

Earthquake

Initial tsunami

Acoustic wave (Longitudinal wave)

Ionospheric thin layer

recombination

GPS receiver
A possible solution

- Tsunami Ionospheric Hole (TIH)
- Slant Total Electron Content (TEC)
- Initial tsunami
- Earthquake
- GPS receiver
- Ionospheric thin layer
- 300km
- Sea
- GPS satellite
A possible solution
A possible solution

- Initial tsunami
- TEC depression
- Acoustic wave
- GPS data
- Statistics
Our approach

- Data preprocessing
  - Low pass filter
  - Outlier detection

- Surface fitting
  - GP regression
  - Uncertainty evaluation

- Analysis
  - Depression propagation
  - Overlapping with tsunami

- Proposal for a new measure
  - Volume of TIH
  - Early warning system
[Redisplaying] Detected data

Ionospheric thin layer

300km

GPS receiver

Tsunami Ionospheric Hole (TIH)

GPS satellite

Slant Total Electron Content (TEC)

Initial tsunami

Earthquake

θ
Detected data

**Satellite 26**
This satellite is the one which is located near the epicenter. All the graphs displayed here are electron density depression data captured by the satellite 26. The data can be purchased by the Japanese government.
The Fourier Transform transforms a function $f$ into a function $\hat{f}$ that describes the frequencies contained in $f$. In other words, the Fourier Transform is a frequency domain representation of $f$.

\[
\hat{f}(\xi) := \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} \, dx
\]
Detected data

Satellite 26
This satellite is the one which is located near the epicenter. All the graphs displayed here are electron density depression data captured by the satellite 26. The data can be purchased by the Japanese government.
The frequency of the ionospheric oscillation is consistent with that predicted by numerical models of acoustic resonance between the ground surface and the lower thermosphere.

(Reference)
Low pass filter

- The data shows the electron density depression.
- Many fluctuations are observed.
- High frequency components should be removed.

- **Fouire Taransform**  
  Not applicable to Real-time analysis

- **Linear Regression**  
  Not applicable to Real-time analysis

- **Backward Moving Average**  
  Applicable to Real-time analysis
Low-pass filtered data

Only low frequency modes can be obtained through the low-pass filter based on the backward moving average.
Our approach

Data preprocessing
- Low pass filter
- Outlier detection

Surface fitting
- GP regression
- Uncertainty evaluation

Analysis
- Depression propagation
- Overlapping with tsunami

Proposal for a new measure
- Volume of TIH
- Early warning system
[Redisplaying] Detected data

- **Ionospheric thin layer**
- **GPS receiver**
- **Tsunami Ionospheric Hole (TIH)**
- **θ**
- **Slant Total Electron Content (TEC)**
- **Initial tsunami**
- **Earthquake**
- **GPS satellite**

300 km
Outliers

3D plot of low-pass filtered data

These two data points seem to be outlier
Outlier Detection

1. Original Data (Low-pass filtered)
2. Projecting onto z axis
3. Outlier detection based on KNN-method
Previous method

Original Data (Low-pass filtered)

Projecting onto z axis

This point can be detected as outlier

TECu
Previous method

Projecting onto z axis

Original Data (Low-pass filtered)

This point cannot be detected as outlier
New method

Dividing original data based on the distance

This point can be detected as an outlier
Outlier Detection

**Input:** Threshold = 3,

- $n$: The number of the observation points in the targeting area

> for $i = 1, \ldots, n$ do
> Select a data point $S_i$ from the entire targeting area
> Select all data points $\{T\}$ located within 200km radius circle centered on $S_i$
> Count the number, $n_i$, of $\{T\}$ and set $k_i = \sqrt{n_i}$
> for $j = 1, \ldots, n_i$ do
> Calculate the distance $d_j$ between $T_j$ and $S_i$
> end for
> Sort $\{T\}$ based on $\{d\}$ in ascending order
> Calculate the TEC difference $D_i$ between $S_i$ and the $k_i$th nearest point
> if $D_i > 3$ then
> $S_i$ is considered as an outlier
> Identify the receiver which detects $S_i$
> end if
> end for

The method can be implemented in real-time
Variogram Cloud (at 6:08:00 UTC)

- Two outliers are detected by the method.
- These two points make the variogram cloud strange.

\[ y = \frac{|\text{TEC}(x_1) - \text{TEC}(x_2)|^2}{2} \]
\[ x = |x_1 - x_2| \]
Outliers and Broken Receivers

These two outliers are detected by receiver 175 and 588.

- GPS receiver 175
- GPS receiver 588

were broken and repaired by the Japanese government organization.

- Getting distorted can be avoided
- Broken receivers can be identified.
Original Data Captured by Satellites

Original data from 5:30:00 to 6:16:30

The red star mark ★ is the epicenter

The earthquake occurred at 6:46:30
Problems

Data points are not uniformly distributed.

The electron density depression is anisotropic.

Detected measurement points are moving.

Network is sparse in some areas (other than Japan).
Our approach

Data preprocessing
- Low pass filter
- Outlier detection

Surface fitting
- GP regression
- Uncertainty evaluation

Analysis
- Depression propagation
- Overlapping with tsunami

Proposal for a new measure
- Volume of TIH
- Early warning system
Gaussian Process Regression

Matérn Kernel covariance function

\[
k_\nu(x_p, x_q) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left( \frac{\sqrt{2\nu r}}{l} \right)^\nu K_\nu \left( \frac{\sqrt{2\nu r}}{l} \right), \quad \text{where } r = |x_p - x_q| \\
\text{cov}(y_p, y_q) = k_\nu(x_p, x_q) + \sigma^2 \delta_{p,q} 
\]

The smoothness depends on \( \nu \).

Advantages

- Nonparametric method
- Evaluation of the uncertainty
Surface Fitting

- The TEC depression (TIH) is estimated successfully.
- 2D projection shows that TIH has anisotropy.
Sparse Data

Remove **95%** of GPS receivers.
Apply our method to the sparse data

TIH is estimated successfully

Our method can be applied to data in other countries where GPS receiver network is sparse.
### Computation time

<table>
<thead>
<tr>
<th>Time</th>
<th>5:46:30</th>
<th>6:00:00</th>
<th>6:08:00</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Computation time (sec)</td>
<td>411.5</td>
<td>627.6</td>
<td>653.6</td>
<td>736.7</td>
</tr>
</tbody>
</table>

Data is detected by satellite every 30 seconds.

Computation time is more than 10 minutes.

Acceleration is necessary to apply the method in real-time.
Computation acceleration

**Ordinary Gaussian Process**
- Cholesky decomposition
  ---\( \text{Computation burden } O(n^3) \)

**INLA-SPDE approach**
- inference with a GMRF
  ---\( \text{Computation burden } O(n^{3/2}) \)

---

**GF**
Gaussian Field (GF) with Matérn covariance function represents the Electron density depression.

**SPDE**
A certain stochastic partial differential equation (SPDE) can construct Gaussian Markov Random Field (GMRF).

**GMRF**
GMRF is defined by sparse matrices that allow for computationally effective numerical methods.

**INLA**
Integrated Nested Laplace Approximation (INLA) algorithm can deal with Bayesian inference for GMRF.

**Fitting**
Computation for the surface fitting can be accelerated.
This approach, that is the SPDE approach, uses a finite element representation to define the GF with Matérn function as a linear combination of basis functions defined on a triangulation of the domain D.

INLA-SPDE approach

The basis function representation

\[ x_{GF}(u) = \sum_{i=1}^{n} \psi_i(u) \omega_i \]  (2)

- \( \psi_i(u) \) : basis functions
- \( \omega_i \) : weights
- \( n \) : the total number of vertices,

- The height of each triangle is given by the weight.
- The values in the interior of the triangle are determined by linear interpolation.

Gaussian Markov Random Fields (GMRF)

A spatial process that models the spatial dependence of data observed on areal units, such as regular grid, lattice structure or geographic regions and has Markovian property (See the next slide sheet).

- An n-dimensional GMRF with mean $\mu$ and symmetric and positive definite precision matrix $Q$, is expressed as

$$x_{GMRF} \sim \mathcal{N}(\mu, Q^{-1})$$

- The density of $x_{GMRF}$ is

$$\pi(x_{GMRF}) = (2\pi)^{-n/2} |Q|^{1/2} \exp \left[ -\frac{1}{2} (x_{GMRF} - \mu)^T Q (x_{GMRF} - \mu) \right]$$

The Markovian property is related to the definition of a neighbourhood structure, in that the full conditional distribution of $x_i$ ($i = 1, \ldots, n$) depends only on a few of the components of $x$.

The conditional distribution of $x_i$

$$\pi(x_i|x_{-i}) = \pi(x_i|x_{\delta_i})$$

The notation $x_{-i}$ denotes all elements in $x$ except for $x_i$

This conditional independence relation can be written as

$$x_i \perp x_{-\{i,\delta_i\}} \mid x_{\delta_i}$$

The key point is that this conditional independence property is strictly related to the precision matrix $Q$.

INLA-SPDE approach

the precision matrix $Q$.

Sparsity $j \notin \{i, \delta_i\} \iff Q_{i,j} = 0$

The nonzero pattern of $Q$ is given by the neighbourhood structure of the process.

In other words, $Q_{i,j} \neq 0$ if $j \in \{i, \delta_i\}$

Ordinary matrix factorization for a dense matrix $O(n^3)$

Temporal GMRF with the sparse matrix $O(n)$

Spatial GMRF with the sparse matrix $O(n^{3/2})$

Spatio-temporal GMRF with the sparse matrix $O(n^2)$

A stochastic weak solution to the SPDE is given by requiring that

\[
\begin{align*}
\langle \varphi_i, (\kappa^2 - \Delta)\alpha/2 \mathbf{x}_{GF} \rangle & \overset{D}{=} \langle \varphi_i, \nu \rangle, \\
\langle \varphi_i, (\kappa^2 - \Delta)\alpha/2 \psi_l \rangle & \overset{D}{=} \langle \varphi_j, \nu \rangle
\end{align*}
\]

for each set of test functions \( \varphi \)

When \( \alpha = 2 \), \( \varphi = \psi \)

\[
\begin{align*}
\langle \psi_i, (\kappa^2 - \Delta) \psi_l \rangle & \overset{D}{=} \langle \psi_j, \nu \rangle, \\
(\kappa^2 \langle \psi_i, \psi_l \rangle + \langle \psi_i, -\Delta \psi_l \rangle) & \overset{D}{=} \langle \psi_j, \nu \rangle
\end{align*}
\]

\[C \quad G \quad \mathcal{N}(0, C)\]

**INLA-SPDE approach**

A stochastic weak solution to the SPDE is given by

\[
\mathbf{x}_{GF}(u) = \sum_{i=1}^{n} \psi_i(u) \omega_i \tag{2}
\]

, where \((\kappa^2 C + G)\omega \sim \mathcal{N}(0, C)\)

The precision of the weight, \(\omega\), is

\[(\kappa^2 C + G)^T C^{-1} (\kappa^2 C + G)\]

As you know, \(C\) and \(G\) are sparse, but \(C^{-1}\) is not.

To obtain sparse precision matrix we replace the \(C^{-1}\) matrix with a diagonal matrix \(\tilde{C}^{-1}\) with elements

\[
\tilde{C}_{i,i} = \int \psi_i(s) ds
\]

The solution has the Markovian property

D. Simpson, F. Lindgren, H. and Rue, "In order to make spatial statistics computationally feasible, we need to forget about the covariance function." Environmetrics 23.1 65-74 (2012)

Bayesian inference

It is possible to make use of the Integrated Nested Laplace Approximation (INLA) algorithm. It produces almost immediately accurate approximations to posterior distributions

$$\pi(x|y) \propto \pi(y|x)\pi(x)$$

$$\pi(x, \theta|y) \propto \pi(\theta)\pi(x|\theta)\pi(x) \prod \pi(y_i|x_i, \theta)$$

$$\tilde{\pi}_{LA}(x_i|\theta, y) \propto \frac{\pi(x, \theta, y)}{\tilde{\pi}_{GC}(x_{-i}|x_i, \theta, y)} \bigg|_{x_{-i} = x^*_{-i}(x_i, \theta)}$$

$$\tilde{\pi}_{GC}(x_{-i}|x_i, \theta, y)$$ is a Gaussian approximation to

$$x_{-i}|x_i, \theta, y$$ around its mode $$x^*_{-i}(x_i, \theta)$$

$$x$$ is assumed to be a Gaussian Markov random field

$$Q(\theta)$$ is variance-covariance matrix

Observations $$y_i$$ are independent of each other
Discretization with triangulation

The number of mesh: 4525

Need to find
1. the number mesh elements which allows faster computation and appropriate fitting.
2. the number threads to parallelise for faster computation.
Acceleration

The number of mesh elements

If the number of mesh elements is smaller than around 4500, the surface fitting fails. Also, the computation burden is $\mathcal{O}(n^{3/2})$.

Parallelization efficiency depends on the number of mesh elements. The most efficient number cannot be known until you do the actual calculations.
Parallelization

The number of mesh elements
The number of threads

If the number of mesh elements is smaller than around 4500, the surface fitting fails. Also, the computation burden is $\frac{3^2}{2}$. Parallelization efficiency depends on the number of mesh elements. The most efficient number cannot be known until you do the actual calculations.

Strange Spike

TeCu

Computational time (Second)

Latitude

Longitude

6-06-00

Strange Spike
Accelerated computation

<table>
<thead>
<tr>
<th>Time</th>
<th>5:46:30</th>
<th>6:00:00</th>
<th>6:08:00</th>
<th>6:16:00</th>
</tr>
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<tbody>
<tr>
<td>Ordinary Gaussian Process</td>
<td>411.5</td>
<td>627.6</td>
<td>653.6</td>
<td>736.7</td>
</tr>
<tr>
<td>INLA-SPDE</td>
<td>51.9</td>
<td>50.0</td>
<td>48.2</td>
<td>49.9</td>
</tr>
</tbody>
</table>

Computation time is less than 1 minute

Real-time analysis
1 Background

2 Application of statistics

3 Results

4 Future step
Anisotropic property

Our new method

Previous study 1

Previous study 2

Anisotropy
Observation/our method

Symmetry
Simulation output
Expansion in each direction

Original data can’t show the expansion speed in each direction because of the data limitation.

Our method succeeds in estimating expansion speed in 8 directions for the first time.
Saito, T. et al., Tsunami source of the 2011 Tohoku-Oki earthquake, Japan: Inversion analysis based on dispersive tsunami simulations, Geophysical Research Letters, 38, (2011)

**Initial tsunami** area estimated by another research group overlaps the **electron density depression** area.

It indicates that the information obtained from the **electron density depression** contains the information about the **initial tsunami**.
Initial tsunami information

- TEC depression
- Initial tsunami region
- Initial tsunami height?

Input data for tsunami propagation model.
E.g. Jugrus, VOLNA....
Initial tsunami information

TEC depression

Initial tsunami
Initial tsunami information

Initial tsunami shape and TEC depression shape are correlated.
Initial tsunami information

TEC depression

Turn it upside down

Initial tsunami shape and TEC depression shape are correlated.
Initial tsunami information

Explicitly TEC depression data has information about initial tsunami shape
Volume computation and warning system

The earthquake occurrence

Threshold

Time

Volume

Fitting surface (full data)
80% CI (full data)
99% CI (full data)
Fitting surface (sparse data)
80% CI (sparse data)
99% CI (sparse data)
Volume = 20 (Threshold)

The depression time series

6:03:00

6:08:00

6:13:00
Our New Method

Frequency Filter
Outlier detection
Surface fitting
Computation Acceleration

- 1 -
- 2 -
- 3 -
- 4 -

✔ Dense estimation with uncertainty
✔ Propagation speed estimation
✔ Tsunami early warning system
✔ Initial tsunami information possibility

Ultimate goal

Tsunami propagation model

Acoustic wave Propagation model

Inverse Estimation

TEC depression

Satellite detection

Ionosphere
Initial tsunami
Initial Tsunami

- **More than 0.5 [m]**
  - **Longitude: 142.555**
    - Height: 2.49 [m]
    - Distance: 120.8 [km]
  - **Longitude: 142.755**
    - Height: 3.38 [m]
    - Distance: 150.7 [km]
Initial Tsunami

More than 0.5 [m]

Longitude: 142.955

4.21 [m]
288.0 [km]

More than 0.5 [m]

Longitude: 143.155

6.06 [m]
342.7 [km]
Initial Tsunami

More than 0.5 [m]

Longitude: 143.355

6.62 [m]
344.7 [km]

Longitude: 143.555

7.24 [m]
336.0 [km]
Initial Tsunami

More than 0.5 [m]

Longitude: 143.755

8.36 [m]
368.7 [km]

Longitude: 143.955

6.59 [m]
378.2 [km]
Initial Tsunami

More than 0.5 [m]

Longitude: 144.155

3.30 [m]  
190.3 [km]

Longitude: 144.355

2.30 [m]  
84.6 [km]
Ultimate goal

Acoustic wave propagation model

Inverse estimation

Satellite detection

TEC depression

Tsunami propagation model

Tsunami

Ionosphere
Ultimate goal

Ionosphere

Acoustic wave propagation model

Inverse estimation

TEC depression

Satellite detection

Tsunami propagation model

Tsunami
Ultimate goal

Ionosphere

TEC depression

Satellite detection

Acoustic wave propagation model

Inverse estimation

Tsunami

Tsunami propagation model
The concept of History Matching involves matching the model output to observations. The implausibility of a parameter subspace is determined by the equation:

\[ I(\theta) = \frac{\| z - E(\eta(\theta)) \|}{\sqrt{\text{Var}(z - E(\eta(\theta)))}} \]

where:
- \( z \): Observation
- \( E(\eta(\theta)) \): Expectation of the model output

If the implausibility exceeds a certain threshold, the input parameter \( \theta \) must be implausible.
Toy Example (1 dimension)

1 dimensional example

Observation = -0.8
Observation error = 0.0004

Simulation model

\( f_{sim}(\theta) \)
This simulation model has high computational cost.
Only 6 points are computed.
Toy Example (run a simulation model)

1 dimensional example

Observation = -0.8
Observation error = 0.0004
Toy Example (run a simulation model)

1 dimensional example

Observation = -0.8
Observation error = 0.0004
1 dimensional example

Observation = -0.8
Observation error = 0.0004

Emulator

Gaussian process
Matern Kernel

\[ I(\theta) = \frac{\| z - E(\eta(\theta)) \| }{\sqrt{\text{Var} \left( z - E(\eta(\theta)) \right) }} \]
Toy Example (compute the implausibility)

1 dimensional example

Observation = -0.8
Observation error = 0.0004

Emulator

Gaussian process
Matern Kernel

Implausibility

\[ I(\theta) = \frac{||z - E(\eta(\theta))||}{\sqrt{\text{Var} \left( z - E(\eta(\theta)) \right)}} \]
1 dimensional example

Observation = -0.8
Observation error = 0.0004

Emulator

Gaussian process
Matern Kernel

$\text{Observation} = -0.8$
$\text{Observation error} = 0.0004$

$\text{Gaussian process}$
$\text{Matern Kernel}$

**Implausibility**

$$I(\theta) = \frac{\| z - E(\eta(\theta)) \|}{\sqrt{\text{Var} \left( z - E(\eta(\theta)) \right)}}$$

Randomly choose an additional point to simulate
Toy Example (an additional point)

1 dimensional example

Observation = -0.8
Observation error = 0.0004

Emulator

Gaussian process
Matern Kernel

Implausibility

\[ I(\theta) = \frac{\|z - E(\eta(\theta))\|}{\sqrt{\text{Var}(z - E(\eta(\theta)))}} \]

Randomly choose an additional point to simulate
1 dimensional example

Observation = -0.8
Observation error = 0.0004

Emulator

Gaussian process
Matern Kernel

Implausibility

\[ I(\theta) = \frac{\|z - E(\eta(\theta))\|}{\sqrt{\text{Var}(z - E(\eta(\theta)))}} \]
Thank you for your listening!

[Our result]

R. Kanai, M. Kamogawa, T. Nagao, A. Smith, and S. Guill "Robust uncertainty quantification of the volume of tsunami ionospheric holes for the 2011 Tohoku-Oki Earthquake: towards low-cost satellite-based tsunami warning systems" (preprint)

[Acoustic wave propagation]


[SPDE approach]

- D. Simpson, F. Lindgren, H. and Rue, "In order to make spatial statistics computationally feasible, we need to forget about the covariance function." Environmetrics 23.1 65-74 (2012)