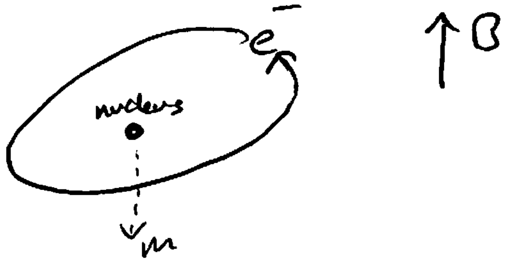
Magnetic Properties of Materials.
Solutions to Problem Sheet

John Morton 05/2010

Q1

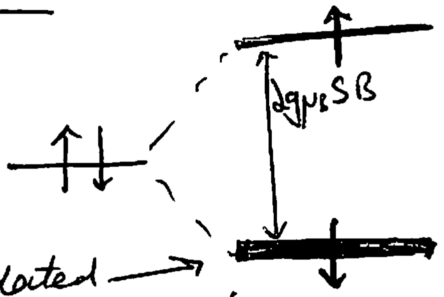
a) Diamagnetism



- Lenz's Law

electrons orbit to oppose magnetic field applied, creating a current loop and thus magnetic moment.

b) Paramagnetism

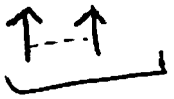


Energy levels for spin-up and spin-down split under applied B field.

More populated due to Boltzmann dist.

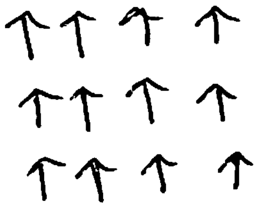
∴ net magnetic moment.

c) Ferromagnetism



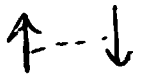
strong interaction between spins leads to alignment

$J > 0$

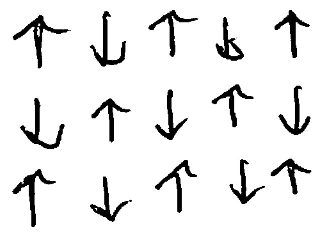


magnetic ordering all spins aligned, below T_c : Curie temperature.

d) Anti-ferromagnetism

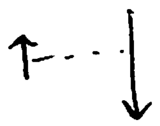


J (exchange interaction) is negative.



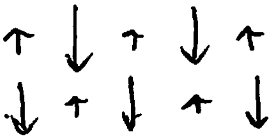
magnetic ordering, but zero net magnetisation, as spins anti-align.

e) Ferri-magnetism



Anti-ferromagnetic coupling between spins of different sizes.

∴ net magnetisation



To characterise solid sample...

① Measure magnetisation M using Force method, or VSM, or SQUID

② Plot M vs. magnetising field H to get χ, susceptibility.

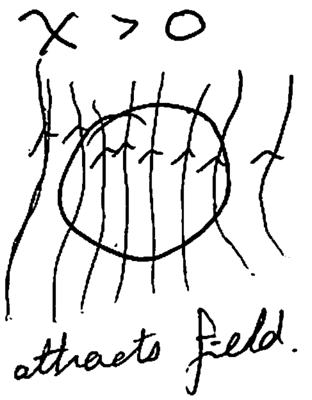
③ Plot $1/\chi$ vs. Temperature.

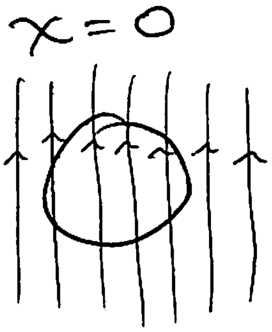
- Negative χ → Diamagnet.
- Straight line, with intercept at $T=0$ → Paramagnetic
- " with intercept at $T>0$ → Ferromagnetic or Ferrimagnet.
- " " $T<0$ → Anti-ferromagnet.

- (Distinguish Ferro- from Ferri-magnet by extracting effective magnetic moment and comparing to moment of the ions etc. involved in the structure)

Q2

a)





Both $\chi > 0$ and $\chi < 0$ lead to inhomogeneous magnetic field.

b) $Cu^+ : [Ar]\,3d^{10}$ $\therefore$ No unpaired electrons

$\Rightarrow$ No Curie paramagnetism.

BUT metal $\therefore$ Pauli paramagnetism.

Also, like everything, some diamagnetism.

$$\chi_{total} = \chi_{Pauli} + \chi_{dia}.$$

$$\chi_{Pauli} = \frac{3}{2}\frac{N\mu_B^2\mu_0}{k_B T_F} = \frac{3}{2}\frac{N\mu_B^2\mu_0}{E_F}$$

Number density of Cu^+ ions $= \dfrac{8960\ kg\,m^{-3}}{63.55\ a.m.u.} = 8.5 \cdot 10^{28}\ /m^3$

Each Cu^+ gives one free electron $\therefore$ $N_{e^-} = 8.5 \cdot 10^{28}\ /m^3$

$\therefore$ $\chi_{pauli} = +1.2 \times 10^{-5}$

$$\chi_{dia} = \frac{N Z \mu_0 e^2 \langle r^2 \rangle}{4 m_e}$$

$Z = 29$
$\langle r^2 \rangle = 0.32\ \mathring{A}$

$\Rightarrow$ $\chi_{dia} = -2.2 \times 10^{-5}$

$\therefore$ $\chi_{total} = -1 \times 10^{-5}$ (net diamagnetic)

c) Need to add something paramagnetic ($\chi > 0$) to compensate.

d) $Rh^{3+} \rightarrow [Kr]4d^6 \Rightarrow$ $S=2$ ground state

or $m = 4\mu_B$.

$\therefore$ Paramagnetic.

$$\chi_{curie} = \frac{N m^2 \mu_0}{k_B T}$$

- Assume $T = 300k$, and neglect diamagnetic contribution

We need this to equal 10^{-5} to cancel out diamagnetis of Cu^+

$$\therefore \frac{16 N \mu_B^2 \mu_0}{k_B T} = 10^{-5} \Rightarrow N = 2.4 \times 10^{25}\ m^{-3}$$

$= \underline{0.03\% \text{ vs. Copper}}$.

e) At lower temperatures, χ_{curie} would rise and the wire would obtain $\chi > 0$, and lose its advantageous behaviour.

Q3 Exchange energy

Positive exchange interaction favours alignment of neighbouring spins.

$U_{ex} = -2J S_1 \cdot S_2$ ∴ goes as $\cos\theta$,

where θ is angle : ↑ ↗ θ

Magnetostatic energy

Total energy stored in the magnetic field, inside and outside the material.

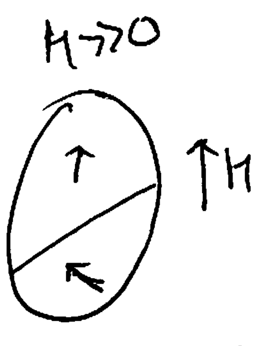
$U = \frac{1}{2}\mu_0 H^2$ per unit volume.

Anisotropy energy

Energy minimised when magnetisation is oriented along certain "easy" axes, as defined by the crystal structure.

a) Exchange energy
b) Magnetostatic energy
c) Anisotropy energy
d) Exchange energy.

domain wall moves.

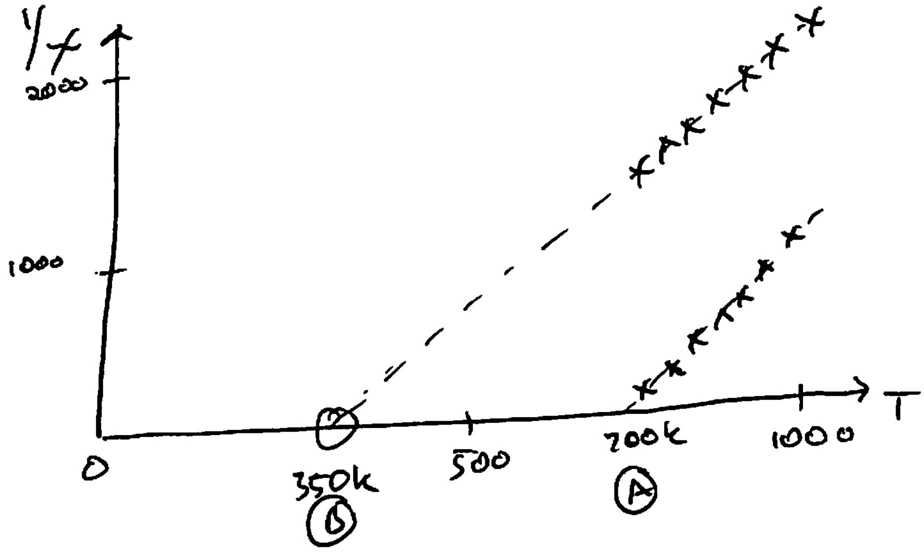


domain magnetisation rotates.

Domain wall motion reversible in very pure materials. Impurities cause pinning of domain walls ⇒ irreversible

Magnetisation rotation: following Stoner-Wohlfahrt model, can be reversible (for field perp. to easy axis), otherwise irreversible.

Q4



T_c (Material A) = 700k

" " B = 350k

$J(B) \approx 2k_BT_c(B) = 9.7\times10^{-21}$ J = 0.06 eV

$J(A) = 1.9\times10^{-20}$ J = 0.12 eV

Domain wall thickness $w = \pi S\sqrt{\frac{J}{Ka}}$

$w(A) = 20$nm ⇒ $K(A) = 4\times10^5$ J/m^3 ← "hard"

$w(B) = 200$nm ⇒ $K(B) = 2\times10^3$ J/m^3 ← "soft"

Q5

a) Electromagnet cores

* Large B_s (eg. $B_s = 2T$)

To turn a modest current into the maximum possible magnetic field.

b) Transformer cores

[high B_s] - - - {high μ} [low B_c] - - -

$\longleftarrow$ Low fequency ——— High fequency $\longrightarrow$

Important for...

(Also want low resistance).

e.g. Mumetal $B_c = 0.25 \mu T$
μ = several 10^5

c) Permanent magnets

Large B_s, but especially B_r (B_r hopefully close to B_s).

e.g. $B_r = 1.8T$

d) Information storage

Moderate B_c (so info. isn't erased) e.g. 20 mT - 200 mT

Moderate B_s e.g. 0.5 T.

Other factors: Cost, formability, corrosion resistance ...