Problems

Q.1 Ionic conductivity

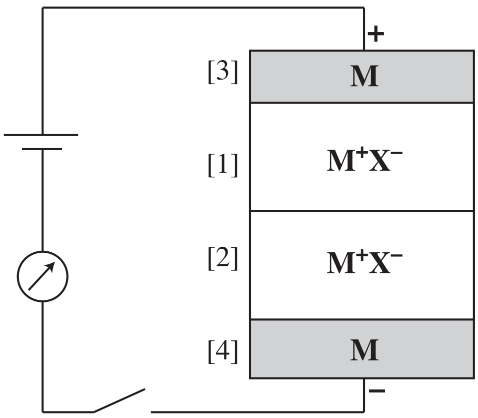
The conductivity σ of an alkali halide crystal containing a small amount of the same halide of a divalent metal varies with temperature T in the following way:

T/K	σ (Sm^{-1})	T/K	σ (Sm^{-1})
300	1.35×10^{-7}	800	4.39×10^{-2}
400	1.27×10^{-5}	900	2.09×10^{-1}
500	1.86×10^{-4}	1000	9.37×10^{-1}
600	1.20×10^{-3}	1100	2.97
700	6.95×10^{-3}		

Deduce as much as you can from these data.

Q.2

An experiment is set up as depicted in the figure below to deduce the relative contributions of Na^+ and Cl^- ions to the conductivity of NaCl.



After passing a current of 1 mA for 10 hours, the four pieces are separately weighed. Piece [2] has decreased in mass by 2.2 mg, while [1] has grown by 2.2 mg. Piece [3] has decreased in mass by 8.6 mg while [4] has grown by 8.6 mg. Find the relative contributions of Na^+ and Cl^- ions to the conductivity.

Q.3 Superconductivity

You have been given a large grant from a recent government stimulus package to design a system of levitating skateboards, for individual public transportation over short distance. The finance minister has shares in a large tin mine and so has required that you perform your initial design study using this material.

a) What type of superconductor is tin?

b) What is the critical current through a wire of tin with 1 mm radius at 2 K? The critical field of tin at 2 K is 20 mT.

Consider a small piece S of tin below T_c (with volume V and density ρ) which is placed directly at some height r above a wire with current I. The magnetostatic force on S is equal to:

$$F = m\frac{\partial B}{\partial r},$$

where m is the total magnetic moment(= volume × magnetisation M), and $\frac{\partial B}{\partial r}$ is the magnetic field gradient along r.

c) Calculate the magnetic field at S, the magnetic field gradient $\frac{\partial B}{\partial r}$, and the magnetic moment of S.

d) By equating the gravitational and magnetostatic forces on S, find a relation between the current through the wire I and the equilibrium height of S above the wire.

e) If the wire is made of tin and kept at 2 K, how high could one levitate S if it's density is 5000 kgm^{-3}.

f) Is this equilibrium position stable? If one had more freedom in design choices, what might you do, and why? Give as many details as you can think of.

Q.4 Dielectric materials

Discuss the physical mechanisms which contribute to the polarisability of dielectric materials.

Q.5

The static relative permittivity ϵ_r of CO_2 and NH_3 is measured at 273 K and 373 K at a pressure of 10^5 Pa (1 atm) and the values are found to be:

	CO_2	NH_3
273 K	1.000988	1.00834
373 K	1.000723	1.00487

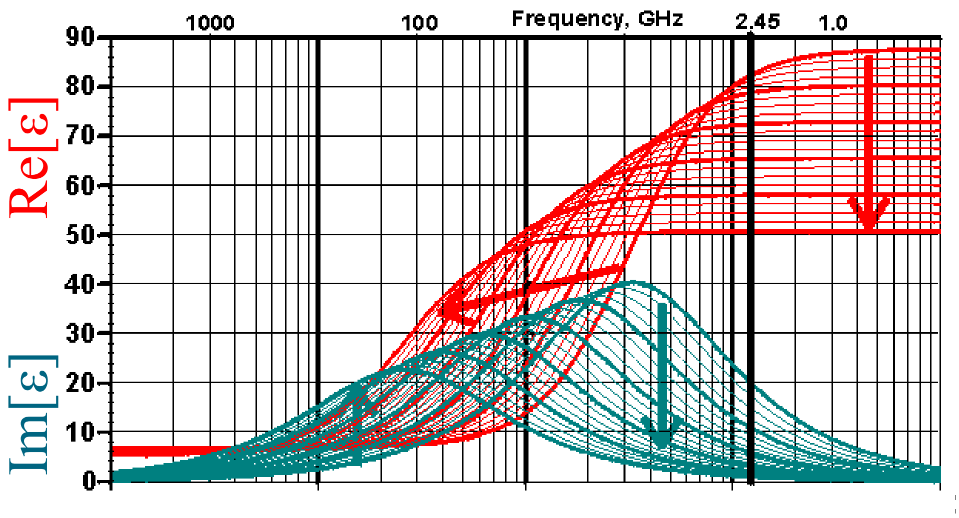
Calculate the permanent electric dipole moment for each gas, and also the radius of the molecule, assuming the electronic polarisability to be the same as that of a conducting sphere ($\alpha = 4\pi\epsilon_0 a^3$).

Note: the volume of 1 mole of gas at 1 atm and 273 K is 22.4 litres.

Q.6

a) Estimate the dipole moment of a water molecule given that the static dielectric constant of water at room temperature is 80 and that the refractive index of water for visible light is 1.33.

b) The figure below plots measurements of the relative permittivity of water as a function of frequency and temperature (0° to 100°C; arrows show increasing temperature). Suggest how such a plot might have been obtained and explain the principle features. What mechanisms are responsible for the frequency and temperature dependence? Extract quantitative information on the physical properties of water. Microwave ovens typically operate at 2.45 GHz. Why might this be a suitable choice for heating food?



Q.7 Ferroelectrics

Two atoms, separated by a distance r, each have polarisability α. Find the relationship between r and α for this system to be ferroelectric. Show also that the condition for a line of atoms with separation r and polarizability α to be ferroelectric is:

$$\alpha > \left(\frac{\pi \epsilon_0 r^3}{\sum_{n=1}^{\infty} n^{-3}} \right)$$

where $\sum n^{-3} = 1 + 1/8 + 1/27 + ... = 1.202$. [Hint: the dipolar field is strongest along the axis of the dipole].

Q.8 Piezoelectrics

Discuss the phenomenon of piezoelectricity. Describe an experimental method of investigating it for quartz. A lighter uses a piezoelectric crystal to produce the spark. The crystal is contained between metal plates which are struck by a hammer. The voltage developed across the plates is applied to a spark gap of breakdown potential 300 V. Calculate the minimum impact velocity of the hammer required to produce the spark, given that its mass is 0.05 kg and that it is brought to rest uniformly in a time of 5 ms.

[The piezoelectric modulus of the crystal in compression $= 3 \times 10^{-10}$ CN^{-1}. The capacitance of the crystal and plates = 10 pF.]

Q.9 Pyroelectrics

In 2005, a table-top fusion experiment was reported where deuterium nuclei were accelerated into a target using the pyroelectric effect in lithium tantalate. The pyroelectric coefficient for lithium tantalate is 5×10^{-6} $\mathrm{CK}^{-1}\mathrm{m}^{-2}$.

In the experiment a single crystal of lithium tantalate was first cooled to 240 K in a sealed chamber, filled with deuterium (^{2}H) gas held at a fixed (low) pressure (10 mTorr). Using a heater, it was subsequently warmed up to 260 K. Estimate change in polarisation in the material arising from the heating process, assuming there are no discharges. Write down the surface charge created by this polarisation.

A grounded copper grid is placed 1 cm from the lithium tantalate crystal. Modelling the system as a charged capacitor, estimate the potential difference and thus electric field between the crystal and the copper grid, assuming $\epsilon_r(^2\mathrm{H}) = 1$. If the breakdown field of deuterium is 10^5 Vm^{-1}Torr, explain what happens and how this produces a source of high-energy deuterium ions.

Q.10 Refractive index

Explain what is meant by the refractive index of a material. One problem with solar light harvesting is that the energy it provides is cyclic across day and night. To overcome this limitation, the government is considering investing in a scheme where alternate solar panels are covered with a 1 cm peice of material with a sufficiently high refractive index that the light takes 12 hours to pass through. What would the required refractive index of such a material be? If such a material existed, would the scheme work? [Hint: consider reflection at an interface].

Q.11 Polarised light

Describe how polarised light may be obtained by making use of *three* essentially different properties of materials. Give example of each.

Q.12 Liquid crystal displays

Explain the principles underlying liquid crystal displays and give a detailed account of one particular system.

Q.13 Optical fibres

Discuss the requirements for long-distance communication using optical fibres and discuss how far different combinations of materials and devices meet them.