

Electrical and optical properties of materials

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Part 4: Maxwell's Equations

$$E = f(z - vt) \quad (4.1)$$

$$\frac{\partial E}{\partial z} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial z}, \quad \text{and} \quad \frac{\partial u}{\partial z} = 1 \quad (4.2)$$

$$\frac{\partial^2 E}{\partial z^2} = \frac{\partial^2 f}{\partial u^2} \frac{\partial u}{\partial z} = \frac{\partial^2 f}{\partial u^2} \quad (4.3)$$

$$\frac{\partial E}{\partial t} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial t}, \quad \text{here} \quad \frac{\partial u}{\partial t} = -v \quad (4.4)$$

$$\frac{\partial^2 E}{\partial t^2} = -v \frac{\partial^2 f}{\partial u^2} \frac{\partial u}{\partial t} = v^2 \frac{\partial^2 f}{\partial u^2} \quad (4.5)$$

$$\frac{\partial^2 E}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2} \quad (4.6)$$

$$E = E_0 \cos\left(\frac{2\pi}{\lambda}(z - vt)\right) \quad (4.7)$$

$$E = E_0 \cos(kz - 2\pi ft) \quad (4.8)$$

$$E = E_0 \cos(kz - \omega t) \quad (4.9)$$

$$k^2 E = \frac{1}{v^2} \omega^2 E \quad \text{or} \quad v = \omega/k \quad (4.10)$$

$$(E\delta x - (E + \delta E)\delta x) = \frac{\delta B_y}{\delta t} \delta x \delta z \quad (4.11)$$

$$\frac{\delta E_x}{\delta z} = -\frac{\delta B_y}{\delta t} \quad (4.12)$$

$$(B\delta y - (B + \delta B)\delta y) = \mu J_x \delta y \delta z \quad (4.13)$$

$$\frac{\delta B_y}{\delta z} = -\mu J_x \quad (4.14)$$

4. Maxwell's equations for electromagnetism

$$\frac{\partial B_y}{\partial z} = -\mu \left(J_x + \epsilon \frac{\partial E_x}{\partial t} \right) \quad (4.15)$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z} \frac{\partial B_y}{\partial t} \quad (4.16)$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial t} \frac{\partial B_y}{\partial z} \quad (4.17)$$

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{\partial}{\partial t} \left(\mu_0 \epsilon_0 \frac{\partial E_x}{\partial t} \right) \quad (4.18)$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad (4.19)$$

$$\frac{1}{v^2} = \mu_0 \epsilon_0 \quad \text{or} \quad v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad (4.20)$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1} \quad \text{and} \quad \epsilon_0 = 8.854 \times 10^{-12} \text{ s}^2\text{H}^{-1}\text{m}^{-1} \quad (4.21)$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 299\,792\,458 \text{ ms}^{-1} \quad (4.22)$$

$$c = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{c_0}{\sqrt{\epsilon_r \mu_r}} \quad (4.23)$$

$$n = \frac{c_0}{c} = \sqrt{\epsilon_r \mu_r} \quad (4.24)$$

$$ikE_0 \exp(i(kz - \omega t)) = i\omega B_0 \exp(i(kz - \omega t)) \quad (4.25)$$

$$\frac{E_0}{B_0} = \frac{\omega}{k} = c \quad (4.26)$$

$$Z = \frac{E_x}{H_y} = \frac{\mu E_0}{B_0} = \mu c = \frac{\mu}{\sqrt{\mu \epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \quad (4.27)$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \, \Omega \quad (= 120\pi \, \Omega \text{ is a common approx.}) \quad (4.28)$$

$$Z = \mu_r \mu_0 c = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} \quad (4.29)$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z} \frac{\partial B_y}{\partial t} \quad (4.30)$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial t} \frac{\partial B_y}{\partial z} \quad (4.31)$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu \frac{\partial}{\partial t} \left(J_x + \epsilon \frac{\partial E_x}{\partial t} \right) \quad (4.32)$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu \frac{\partial}{\partial t} \left(\sigma E_x + \epsilon \frac{\partial E_x}{\partial t} \right) \quad (4.33)$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu \sigma \frac{\partial E_x}{\partial t} + \mu \epsilon \frac{\partial^2 E_x}{\partial t^2} \quad (4.34)$$

$$k = \sqrt{\mu \epsilon \omega^2 + i \omega \mu \sigma} \quad (4.35)$$

$$E = E_0 e^{i(kz - \omega t)} = E_0 e^{i(\text{Re}[k]z - \omega t)} e^{-\text{Im}[k]z} \quad (4.36)$$

$$\text{Re}[k] = \text{Im}[k] = \sqrt{\frac{\sigma \mu \omega}{2}} \quad (4.37)$$

$$\delta = \sqrt{\frac{2}{\sigma \mu \omega}} \quad (4.38)$$

$$U = \frac{1}{2} (\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}) = \frac{1}{2} (\epsilon E_x^2 + \mu H_y^2), \quad (4.39)$$

$$cU = \frac{1}{2} \left(\sqrt{\frac{\epsilon}{\mu}} E_x^2 + \sqrt{\frac{\mu}{\epsilon}} H_y^2 \right) = \frac{1}{2} \left(\frac{E_x^2}{Z} + Z H_y^2 \right), \quad (4.40)$$

$$flux = cU = E_x H_y = \frac{E_x^2}{Z} = Z H_y^2 \quad (4.41)$$

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \quad (P_z = E_x H_y) \quad (4.42)$$