Electrical and optical properties of materials

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Part 2: Dielectric properties of materials

$$D = \epsilon_0 E + P \tag{2.1}$$

$$D = \epsilon_0 E + \chi_e \epsilon_0 E = \epsilon_0 (1 + \chi_e) E = \epsilon_0 \epsilon_r E \tag{2.2}$$

$$\epsilon_r = \frac{P}{\epsilon_0 E} + 1 \tag{2.3}$$

$$I = I_R + I_C = \frac{V_0 \sin \omega t}{R} + C V_0 \omega \cos \omega t \tag{2.4}$$

$$\tan \delta = \frac{I_R}{I_C} = \frac{V_0/R}{C V_0 \omega} = \frac{1}{\omega C R} \tag{2.5}$$

$$W = \frac{1}{T} \int_0^T VI = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} V_0 \sin \omega t \left(\frac{V_0 \sin \omega t}{R} + C V_0 \omega \cos \omega t \right) dt \tag{2.6}$$

$$W = \frac{\omega V_0^2}{2\pi} \int_0^{2\pi/\omega} \left(\frac{1 - \cos 2\omega t}{2R} + C\omega \sin \omega t \cos \omega t \right) dt \tag{2.7}$$

$$W = \frac{V_0^2}{2R} = \frac{1}{2} \omega V_0^2 C \tan \delta = \frac{1}{2} \omega V_0^2 C_0 \epsilon_r \tan \delta \tag{2.8}$$

$$Z = \frac{1}{i\omega C} = \frac{1}{i\omega \epsilon_r C_0} \tag{2.9}$$

$$Z = \frac{1}{i\omega C_0 [\mathrm{Re}(\epsilon_r) + i\mathrm{Im}(\epsilon_r)]} = \frac{\mathrm{Re}(\epsilon_r)}{i\omega C_0 |\epsilon_r|^2} - \frac{\mathrm{Im}(\epsilon_r)}{\omega C_0 |\epsilon_r|^2} \tag{2.10}$$

$$C' = \frac{C_0 |\epsilon_r|^2}{\mathrm{Re}(\epsilon_r)}, \quad R = \frac{\mathrm{Im}(\epsilon_r)}{\omega C_0 |\epsilon_r|^2}, \quad \text{and hence} \quad \tan \delta = \frac{1}{\omega C R} = \frac{\mathrm{Im}(\epsilon_r)}{\mathrm{Re}(\epsilon_r)} \tag{2.11}$$

$$P = n \alpha E_{loc}, \tag{2.12}$$

$$\delta E_r = \frac{P \cos \theta}{4\pi \epsilon_0 r^2} \delta A \tag{2.13}$$

$$E_s = \oiint_{\text{surface}} \delta E_r \cos\theta \tag{2.14}$$

$$E_s = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{P\cos^2\theta}{4\pi\epsilon_0 r^2} r^2 \sin\theta \ \delta\theta \ \delta\phi \tag{2.15}$$

$$E_s = \int_{\theta=0}^{\pi} \frac{P\cos^2\theta}{4\pi\epsilon_0 r^2} 2\pi r^2 \sin\theta \ \delta\theta \tag{2.16}$$

$$E_s = \int_{\theta=0}^{\pi} \frac{P\cos^2\theta}{2\epsilon_0} \sin\theta \ \delta\theta \tag{2.17}$$

$$E_s = \frac{P}{2\epsilon_0} \left[\frac{\cos^3\theta}{3}\right]_0^{\pi} \tag{2.18}$$

$$E_s = \frac{P}{3\epsilon_0} \tag{2.19}$$

$$E_{loc} = E + \frac{P}{3\epsilon_0} \tag{2.20}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{n\alpha}{3\epsilon_0} \tag{2.21}$$

$$\delta n = C \exp\left(\frac{-U}{k_B T}\right) \delta U \tag{2.22}$$

$$P = n \frac{\int p\cos\theta \ dn}{\int dn} \tag{2.23}$$

$$P = n \frac{\int_0^{\pi} p\cos\theta \ C\exp\left(\frac{pE_{loc}\cos\theta}{k_B T}\right) \ dU}{\int_0^{\pi} C\exp\left(\frac{pE_{loc}\cos\theta}{k_B T}\right) \ dU} \tag{2.24}$$

$$P = n \frac{\int_0^{\pi} p\cos\theta \ C\exp\left(\frac{pE_{loc}\cos\theta}{k_B T}\right) \ pE_{loc}\sin\theta \ d\theta}{\int_0^{\pi} C\exp\left(\frac{pE_{loc}\cos\theta}{k_B T}\right) \ pE_{loc}\sin\theta \ d\theta} \tag{2.25}$$

$$P = n \frac{\int_1^{-1} x\exp(xy) \ dx}{\int_1^{-1} \exp(xy) \ dx} \tag{2.26}$$

$$P = np\left(\coth y - \frac{1}{y}\right) = npL(y) \tag{2.27}$$

$$P = \frac{np^2}{3k_BT}E_{loc} \tag{2.28}$$

$$m\left(\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x\right) = qE_0\exp(i\omega t) \tag{2.29}$$

$$x = x_0\exp(i\omega t) = \frac{q}{m}\frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma}E_0\exp(i\omega t) \tag{2.30}$$

$$P = np = nqx = \frac{nq^2}{m}\frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma}E_0\exp(i\omega t) \tag{2.31}$$

$$\alpha = \frac{q^2}{m}\frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma} \tag{2.32}$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{nq^2}{3m\epsilon_0}\frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma} \tag{2.33}$$

$$\frac{n\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \approx \frac{\epsilon_r - 1}{3} \tag{2.34}$$

$$\epsilon_r - 1 = \frac{nq^2}{m\epsilon_0}\frac{1}{(\omega_0^2 - \omega^2) + i\omega\gamma} \tag{2.35}$$

$$\mathrm{Re}(\epsilon_r) = 1 + \frac{nq^2}{m\epsilon_0}\frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2} \tag{2.36}$$

$$\mathrm{Im}(\epsilon_r) = -\frac{nq^2}{m\epsilon_0}\frac{\omega\gamma}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2} \tag{2.37}$$

$$\mathrm{Re}(\epsilon_r) = 1 + \frac{nq^2}{2m\omega\epsilon_0}\frac{(\omega_0 - \omega)}{(\omega_0 - \omega)^2 + \gamma^2/4} \tag{2.38}$$

$$\mathrm{Im}(\epsilon_r) = -\frac{nq^2}{2m\epsilon_0}\frac{\gamma/2}{(\omega_0 - \omega)^2 + \gamma^2/4} \tag{2.39}$$

$$\text{If } \omega \to 0, \quad \text{then } \epsilon_r - 1 \to \frac{nq^2}{m\epsilon_0\omega_0^2} \tag{2.40}$$

$$\text{If } \omega \to \infty, \quad \text{then } \epsilon_r - 1 \to 0 \tag{2.41}$$

$$P(t) = P_0\exp(-t/\tau) \tag{2.42}$$

$$f(\omega) = A \int_0^\infty P(t) \exp(-i\omega t) dt \tag{2.43}$$

$$P(t) = P_0 \exp(-t/\tau) \tag{2.44}$$

$$\epsilon_r(\omega) = A \int_0^\infty P_0 \exp\left(-t\left(i\omega + 1/\tau\right)\right) dt \tag{2.45}$$

$$\epsilon_r(\omega) = \frac{AP_0}{1/\tau + i\omega} + B \tag{2.46}$$

$$\epsilon_r(0) = AP_0\tau + B \qquad \text{and} \qquad \epsilon_r(\infty) = B \tag{2.47}$$

$$\frac{\epsilon_r(\omega) - \epsilon_r(\infty)}{\epsilon_r(0) - \epsilon_r(\infty)} = \frac{1}{1 + i\omega\tau} \tag{2.48}$$

$$\mathrm{Re}(\epsilon_r(\omega)) = \epsilon_r(\infty) + \frac{\epsilon_r(0) - \epsilon_r(\infty)}{1 + \omega^2\tau^2} \tag{2.49}$$

$$\mathrm{Im}(\epsilon_r(\omega)) = \omega\tau \frac{\epsilon_r(0) - \epsilon_r(\infty)}{1 + \omega^2\tau^2} \tag{2.50}$$