Q1  

a) Diamagnetism

- Lenz's law
- Electrons orbit to oppose magnetic field applied, creating a current loop and thus magnetic moment.

b) Paramagnetism

- More populated due to Boltzmann dist.
- Net magnetic moment.

Energy levels for spin-up and spin-down split under applied field.

Energies levels for spin-up.

\[ \uparrow \uparrow \uparrow \downarrow \downarrow \]

Energies levels for spin-down.

\[ \uparrow \uparrow \downarrow \downarrow \uparrow \uparrow \]

More populated due to Boltzmann dist.

C) Ferromagnetism

- Strong interaction between spins leads to alignment
- \( I > 0 \)

Magnetic ordering all spins aligned, below \( T_c \) Curie temperature.
a) Anti-ferromagnetism

\[ \uparrow \downarrow \uparrow \downarrow \]

J (exchange interaction) is negative.

\[ \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \]

magnetic ordering, but zero net magnetisation, as spins anti-align.

e) Ferri-magnetism

\[ \uparrow \downarrow \]

Anti-ferromagnetic coupling between spins of different sizes.

\[ \uparrow \downarrow \uparrow \downarrow \uparrow \]

...net magnetisation: \[ \uparrow \downarrow \uparrow \downarrow \uparrow \]

To characterise solid sample...

1. Measure magnetisation M using Force method, or VSM, or SQUID.
2. Plot M vs. magnetic field H to get X, susceptibility.
3. Plot 1/X vs. Temperature.

- Negative T \( \Rightarrow \) diamagnet.
- Straight line, with intercept at \( T=0 \) \( \Rightarrow \) paramagnet.
- ... with intercept at \( T>0 \) \( \Rightarrow \) Ferromagnetic or Ferrimagnet.
- ... \( T<0 \) \( \Rightarrow \) Antiferromagnet.

(Distinguish Ferro- from Ferrimagnet by extracting effective magnetic moment and comparing to moment of the ions etc. involved in the structure)
$\chi > 0$ attracts field.

$\chi = 0$ repels field.

Both $\chi > 0$ and $\chi < 0$ lead to inhomogeneous magnetic field.

b) $\text{Co}^+: [\text{Ar}] 3d^10$.

- No unpaired electrons
  - No Curie paramagnetism.
- Hot metal.
  - Pauli paramagnetism.
- Also, like everything, some diamagnetism.

$\chi_{\text{total}} = \chi_{\text{Pauli}} + \chi_{\text{dia}}.$

$\chi_{\text{Pauli}} = \frac{3}{2} \frac{N \mu_B^2 \mu_0}{K_B T_F} = \frac{3}{2} \frac{N \mu_B^2 \mu_0}{E_F}$

Number density of $\text{Co}^+$ ions $= \frac{8960 \text{ kg m}^{-3}}{63.55 \text{ a.m.u.}} = 8.5 \cdot 10^{28} \text{ m}^{-3}$

Each $\text{Co}^+$ gives one free electron $\Rightarrow N_e = 8.5 \cdot 10^{28} \text{ m}^{-3}$

$\chi_{\text{Pauli}} = +1.2 \times 10^{-5}$

$\chi_{\text{dia}} = \frac{N Z \mu_0 e^4 \langle r^4 \rangle}{4 m_e}$

$Z = 29$  
$\langle r^4 \rangle = 0.32 \text{ \AA}^4$  
$\chi_{\text{dia}} = -2.2 \times 10^{-5}$

$\Rightarrow \chi_{\text{total}} = -1 \times 10^{-5}$ (net diamagnetic)
c) Need to add something paramagnetic \((X > 0)\) to compensate.

d) \(\text{Rh}^{3+} \rightarrow [\text{Kr}]4d^6 \rightarrow S = 2\) ground state
   
   or \(m = 4\nu_2\).

   \[ X_{\text{Curie}} = \frac{N m^2 \mu_0}{k_B T} \]

   • Assume \(T = 300 \text{K}\), and neglect diamagnetic contribution.
   
   We need \(X_{\text{Curie}}\) to equal \(10^5\) to cancel out diamagnetic
   
   of \(\text{Cu}^{+}\)

   \[ \frac{16 \times 16 \times 10^{-5}}{k_B T} = 10^5 \Rightarrow N = 3.4 \times 10^{25} \text{ m}^{-3} \]

   \[ = 0.03\% \text{ vs. Copper} \]

   e) At lower temperatures, \(X_{\text{Curie}}\) would rise and the wire would obtain \(X > 0\), and lose its advantageous behaviors.
Exchange energy

Positive exchange interaction favours alignment of neighbouring spins.

\[ V_{ex} = -2J S_1 \cdot S_2 \quad \text{goes as} \quad \cos \theta, \]

where \( \theta \) is angle: \( \uparrow \) \( \rightarrow \) \( \downarrow \)

Magnetostatic energy

Total energy stored in the magnetic field, inside and outside the material.

\[ U = \frac{1}{2} \mu_0 H^2 \quad \text{per unit volume}. \]

Anisotropy energy

Energy minimised when magnetisation is oriented along certain "easy" axes, as defined by the crystal structure.

a) Exchange energy
b) Magnetostatic energy
c) Anisotropy energy
d) Exchange energy.

\[ H = 0 \]

\[ H > 0 \]

domain wall moves \[ H \rightarrow 0 \]

\[ \uparrow \]

\[ \rightarrow \]

\[ \downarrow \]

domain magnetisation rotates.
Domain wall motion reversible if very pure materials.
Impurities cause pinning of domain walls ⇒ irreversible.
Magnetisation rotation: following Stoner-Wohlfarth model, can be reversible (for field perp. to easy axis), otherwise irreversible.

\[ T_c \text{ (Material A)} = 700 \text{K} \]
\[ T_c \text{ (Material B)} = 350 \text{K} \]

\[ J(A) = 2k_B T_c(A) = 9.7 \times 10^{-21} \text{ J} = 0.06 \text{ eV} \]
\[ J(B) = 1.9 \times 10^{-20} \text{ J} = 0.12 \text{ eV} \]

Domain wall thickness \( w = \pi S \sqrt{\frac{J}{k_B a}} \)

\( w(A) = 20 \text{nm} \Rightarrow K(A) = 4 \times 10^5 \text{ J/m}^3 \leftarrow \text{"hard"} \)
\( w(B) = 200 \text{nm} \Rightarrow K(B) = 2 \times 10^3 \text{ J/m}^3 \leftarrow \text{"soft"} \)
a) Electromagnet cores

* Large \( B_s \) (e.g. \( B_s = 2 \) T)

To turn a modest current into the maximum possible magnetic field.

b) Transformer cores

\[
\begin{align*}
&[\text{high } B_s] \quad [\text{high } \mu] \quad [\text{low } B_c] \\
&[\text{low frequency}] \quad [\text{high frequency}]
\end{align*}
\]

(Also want low resistance).

e.g. MuMetal

\[
\begin{align*}
B_c &= 0.25 \, \mu T \\
\mu &= \text{several 10}^5
\end{align*}
\]

c) Permanent magnets

Large \( B_s \), but especially \( B_r \) (\( B_r \) hopefully close to \( B_s \)).

e.g. \( B_r = 1.8 \) T

d) Information storage

Moderate \( B_c \) (so info. isn't erased) e.g. \( 20 \) mT

Moderate \( B_s \), e.g. \( 0.5 \) T.

Other factors: cost, formability, corrosion resistance...