

The paired comparison design for 21 rectangles.

Twenty-one rectangles are used in the experiment, with the ratios and LRs shown below. The rectangles were chosen a) to have a somewhat wider range than those used by McManus (1980), where the maximum log.ratio was 60; and b) to include the square and the golden sections. With these constraints it was not quite possible to have exactly equal log ratio steps between stimuli, and the step size therefore increases slightly towards the ends of the series, where rectangles anyway appear to be slightly less easy to distinguish.

Rectangle number	Ratio = (width/height)	LR (log ratio) = $100 \cdot \log_{10}(\text{Width/height})$	
1	0.2049	-69.1	
2	0.259	-58.7	
3	0.32	-49.5	
4	0.387	-41.2	
5	0.46	-33.8	
6	0.537	-27	
7	0.618	-20.9	Vertical golden section
8	0.704	-15.3	
9	0.795	-9.97	
10	0.893	-4.93	
11	1	0	Square
12	1.12	4.93	
13	1.258	9.97	
14	1.421	15.3	
15	1.618	20.9	Horizontal golden section
16	1.863	27	
17	2.175	33.8	
18	2.582	41.2	
19	3.125	49.5	
20	3.866	58.7	
21	4.903	69.1	

The design is summarised in the matrix below, in which the # symbols indicate the diagonal. The letters indicate the three types of pairs used: a) all of the eleven odd-numbered stimuli (which include the square and the two golden sections) are included in a complete 11x11 paired comparison design (55 pairs); b) all pairs of rectangles that are within one step of each other (20 pairs); and c) all additional pairs that are within two steps of one another (9 pairs). The design is efficient, the 11x11 matrix broadly surveying the entire region, whereas the one and two step pairs allow a more fine-grained analysis of fairly similar rectangles.

It should be noted that the simple method of summing across rows or columns to obtain preferences for individual stimuli, that was used by McManus (1980), cannot be used in the incomplete paired comparison design. It is therefore necessary to use the regression method outlined in the next section. That technique has the advantage also of allowing standard errors to be calculated for all preferences (and it should be noted that in general these are larger for the even-numbered rectangles in the current design than for the odd-numbered rectangles, because there are fewer pairs in the design which include an even-numbered rectangle).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	#	b	a		a		a		a		a		a		a		a		a		a
2		#	b	c																	
3			#	b	a		a		a		a		a		a		a		a		a
4				#	b	c															
5					#	b	a		a		a		a		a		a		a		a
6						#	b	c													
7							#	b	a		a		a		a		a		a		a
8								#	b	c											
9									#	b	a		a		a		a		a		a
10										#	b	c									
11											#	b	a		a		a		a		a
12												#	b	c							
13													#	b	a		a		a		a
14														#	b	c					
15															#	b	a		a		a
16																#	b	c			
17																	#	b	a		a
18																		#	b	c	
19																			#	b	a
20																				#	b
21																					#

Circular triad analysis. The total number of triads in the design is 184 (of which 165 come from the complete 11x11 comparison embedded within the design). Because this paired comparison design is incomplete, there is no obvious theory for calculating the expected number of circular triads (unlike complete paired comparison designs – see David (1988)). However it is straightforward to carry out a Monte Carlo analysis of random matrices based on the design. A million Monte Carlo replications found that the mean number of circular triads was 45.99 (SD 5.85), with the maximum number of circular triads being 65. The 5th percentile (with ties) was at 36 circular triads, the 1st percentile (with ties) at 30 triads and the 0.1th percentile (with ties) at 24 triads, so that 35, 29 and 23 can be regarded as significant with $p < .05$, $< .01$ and $< .001$ respectively.

Reference List

David, H. A. (1988). *The method of paired comparisons (2nd edition)*. London: Charles Griffin.

McManus, I. C. (1980). The aesthetics of simple figures. *British Journal of Psychology*, 71, 505-524.