EMPIRICAL STUDIES OF THE ARTS, Vol. 15(2) 209-232, 1997

THE GOLDEN SECTION AND THE AESTHETICS OF FORM AND COMPOSITION: A COGNITIVE MODEL

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ABSTRACT

Previous work on the aesthetics of simple figures such as rectangles and triangles, as well as on the aesthetics of color, suggests that although there are clear population level preferences, there are also large individual differences which are temporally stable, and which any adequate theoretical analysis must take into account. Data presented here show similar phenomena in a related problem in composition-where to place an object within the frame of a picture to produce the optimal aesthetic effect. A novel and powerful "method of randomized paired comparisons" first showed that there are overall population level preferences, with objects being placed preferentially at the two golden sections horizontally, and between the two golden sections vertically. As in the studies of simple figures and colors, there are large individual differences. A cognitive model of "sensory aesthetics" is proposed in which continua of any type (space, geometric objects, colors, or whatever), are described categorically, usually in terms of words such as "square," "rectangle," "line," etc., each of which is a fuzzy set. Preference functions are then derived from the union and intersection of the fuzzy set functions, which differ between individuals as their categories differ or as they prefer objects which are prototypical, or are at the boundaries between prototypes. There is therefore wide inter-individual variability.

... I found pleasure in giving my mind to the problem of beauty and proportion ... I defined (material forms) in two classes, those which please the eye because they are beautiful in themselves, and those which do so because they are properly proportioned in relation to something else ...

St. Augustine, Confessions, IV, 14, 15

He taught us to see the beauty of line, shape and proportion enclosed within a surrounding frame.

[Reginald Ginns on Eric Gill] (MacCarthy, 1989)

The golden section rectangle has been influential in experimental aesthetics, not least in providing a clear hypothesis, of a specifically preferred stimulus form, with the problem of designing appropriate empirical studies which do not themselves bias the responses of subjects. However, experimental aesthetics has failed to meet several challenges imposed by this approach. First, there has been a large emphasis upon group results and little on individual differences. Second, there have been few studies looking at the golden section in other contexts than the rectangle. And third, there is a dearth of psychological theory as to why the golden section might have any role, especially from the perspective of cognitive psychology. This article, which broadly divides into two parts, will try partially to remedy these defects. In the first half we review previous studies on individual differences in aesthetic preferences for simple figures, and then describe empirical results on the role of the golden section in a previously unreported and entirely different context, that of the composition of an image in a frame, again showing both group effects and individual differences. In the second, theoretical, part we present a cognitive approach to modeling such results, arguing that they represent attempts by individuals to classify and structure the various continua that they find in the world around them, and that aesthetic preferences represent different, and potentially idiosyncratic, solutions to that problem of classifying and structuring the visual world, thereby allowing individual differences. Insofar, though, as there are naturally or ecologically important ways of structuring those continua, so group preferences will become apparent.

Fechner was the first scientist to explore the possibility of using an experimental methodology for understanding what had previously been the most abstruse branch of philosophy, aesthetics. The human sense of the *beautiful* could be quantified beyond the highly subjective *ex cathedra* introspections of a small, select group of critics. Systematic statistical analysis of large numbers of choices, of *preferences*, made by many individuals allowed a more objective description of aesthetic phenomena. Since the publications of Fechner's three methods, of *choice*, of *production*, and of *use* (Fechner, 1871; Fechner, 1876), a large amount of systematic data has been collected in a range of different situations.

Despite general acceptance of the validity of the enterprise as a whole, there have been many doubts about the specific data, and of their relation to theory. Fechner's major finding, of the golden section's central role in preference for simple geometrical figures such as rectangles, has been criticized at three levels: that the preferences expressed are the result merely of the "demand characteristics" of the experiment (Godkewitsch, 1974; Piehl, 1976); that rectangles

are perhaps a special case, since Fechner himself found no preference for the golden section in data on ellipses (Witmer, 1894); and that Fechner's emphasis upon the golden section might have been unduly influenced by the somewhat eccentric metaphysics of what Arnheim has called "the other Fechner" (Arnheim, 1986).

To psychologists more than a century after Fechner's death, with the overwhelming predominance of cognitive theory in psychology, and the near complete extinction of aesthetics as a major branch of philosophy (although with some hints of a possible renaissance (Scruton, 1987)), the overwhelming impression is one of indifference. Even if the data of experimental aesthetics are accepted, then the lack of any adequate theory beyond mere mystic numerology destroys most interest in the phenomena. As Arnheim has put it:

Just as Fechner's work does not tell us why people prefer the ratio of the golden section to others, so most of the innumerable preference studies carried out since his time tell us deplorably little about what people see when they look at an aesthetic object, what they mean by saying that they like or dislike it, and why they prefer the objects they prefer (Arnheim, 1955).

In this article we will briefly review some of the literature on the golden section, will describe some new data on the place of the golden section in pictorial composition, and will describe a cognitive theory of aesthetic preference for simple stimuli which is applicable not only to simple geometric figures, but also to composition and to colors.

THE GOLDEN SECTION IN EXPERIMENTAL AESTHETICS

The extensive literature on the golden section has been reviewed by many authors (e.g., Arnheim, 1955; Wittkower, 1960; Benjafield, 1985; Green, 1995; Höge, 1995), and only a brief account will be given here. Fechner's original experiment asked subjects to choose which of a range of ten rectangles, differing only in their width-length ratio, they preferred the most and which the least. There was a clear preference, as there was also in a replication of the experiment by Lalo (1908), for the golden section i.e., the rectangle with height-width ratio of 1:1.6180.... The demand characteristics of this experiment have been criticized, in particular because the golden section was intermediate between the two ends of the stimulus range (Godkewitsch, 1974; Piehl, 1976), leading to anchoring effects. To a large extent such criticisms can be removed by using a method of paired comparison, in which a subject only observes two stimuli at a time, and expresses a relative preference between the two. Several studies have used this method (Piehl, 1978; McManus, 1980). Its principal disadvantage is that if a



Figure 1. Preferences in four different studies for rectangles of different shape (expressed as the log₁₀ of the ratio of width to height). The values on the abscissa of 0, 6, 12, 18, 24, 30, 45, and 60 correspond to ratios of 1, 1.148, 1.318, 1.513, 1.737, 1.995, 2.818, and 3.981. The square (\Box) and golden section (ϕ) are indicated. The preference measure is derived from a complete paired comparison experiment. If there are *n* subjects who make preference judgments between all of the m(m-1)/2 pairs of *m* stimuli, and on *q* occasions (out of *n.(m-1)*) prefer stimulus *X* over the other *m-1* stimuli, then *X* is preferred on a proportion *p* of occasions, where p=q/n(m-1). *p* is necessarily in the range 0-1. For ease of interpretation it is better to use a scaled score, *s*, where s=2(p-.5), which is now in the range -1 to +1, and a value of +1 indicates that *X* is always preferred in all comparisons, and a value of 0 indicates that *X* is preferred half the time and unpreferred half the time (i.e., zero preference). From McManus (1980).

reasonable number of stimuli are to be compared then there can be an unmanageable number of pairs for assessment, so that N stimuli result in N.(N-1)/2 pairs. Figure 1 shows the results of four paired comparison studies of rectangle preference (McManus, 1980). The studies differ only in the particular rectangles presented, in two of the cases always being horizontal (landscape) or vertical (portrait), and in the other two cases being different mixtures of horizontal and vertical rectangles. It is clear that the principle peaks are around the golden section, although there are subsidiary peaks near the square as well. The problem with results such as those of Figure 1 is that they fail to show the range of individual differences. The great advantage of the method of paired comparisons is that it allows significance testing of a single subject's results. Figure 2 shows the preferences of six very different subjects, all of which except S.42 are very significantly different from chance. The individual preference functions of

Figure 2 are very stable in time, Figure 3 showing four subjects all of whom repeated a similar experiment after an interval of over two years. Using a similar method, there are also broad population preferences for triangles drawn within golden section rectangles (McManus, 1980), although again there are large individual differences in preference (and Green, 1995, has questioned whether these are strictly 'golden section triangles' at all). Factor analysis suggests that particular types of preference for rectangles are associated with specific preference functions for triangles. A psychological theory of the aesthetics of simple figures must therefore not only explain the broad population trends of Figure 1, but must also account for the range and stability of the individual differences of Figures 2 and 3.

The problem of composition in pictures is central to the act of creation. Gombrich (1984) has described the two stages of picture making as "framing and filling." The frame forms a boundary within which the composition must be constructed, by a consecutive series of decisions about the placing of pigment. Working artists, as well as introductory books on painting and photography, and accounts of architectural and pictorial composition, frequently recommend a golden section placement of an object relative to the surrounding frame, so that the distances to the two sides, or to the top and bottom, are in the ration of 1:1.618 (Anonymous, 1984; Association des Amis de Boscodon, 1987, Carter, 1953; Popham, 1957; Thomas, 1969). We have investigated this task (which in some ways is the two-dimensional equivalent of experiments asking subjects to divide a line at the optimal position; Angier, 1903), by asking subjects to make a series of paired comparison judgements of pictures in each of which a single principle object was placed in different positions relative to the frame. We were particularly concerned in our experiment to produce an experimental design which would allow individual differences to emerge, and which would have the advantages of the method of paired comparisons, but without the excessive number of stimuli that that often necessitates. The "Method of randomised paired comparison" has these features.

METHOD

Twenty-six subjects took part in the experiment (male 14, female 12; age range 14 to 82). Each subject was tested individually, and viewed a series of pairs of "pictures," and made a relative preference judgment on a 6-point scale, according to whether they "very much," "moderately," or "marginally" preferred the stimulus on the right or on the left.

Each picture consisted of a frame 15.2 cm wide by 11.4 cm high (i.e., the height-width ratio most commonly found in works of art (McManus, unpublished)) defining a field in which was placed one of four similarly sized single objects, two of which were realistic, a picture of a boat and of a church, and two were abstract an open rectangle (2.5 cm high by 1.2 cm wide) and a rectangle



Figure 2. Preferences of six individual subjects for rectangles of different shape. Axes as for Figure 1. From McManus (1980).



with the top half white and the lower half black. The boat gave the impression of floating in a featureless sea, and the church of standing on a featureless plain. In a pilot study the focal point, or centroid, of each object was found by subjects indicating the point they regarded as the "visual center."

In each pair the pictures always had the same object, but the position of the focal point of the object was different within the field. Focal points could be at any of thirteen equally spaced vertical positions between .167 and .833 of the distance from top to bottom of the field, and at any of ten equally spaced horizontal positions from the mid-line to .875 of the distance from the left-hand edge to the right-hand edge. For each picture in each pair the horizontal and vertical positions of the object within the field were chosen at random. Figure 4 shows a typical example of such a pair. Each subject saw 130 such pairs of pictures, and made a preference judgment. Subjects were told in their instructions that "Both pictures in each pair are identical, but placed differently within the frame. I would like you to choose which of the pair you find most pleasing...." Subjects were asked to work fairly quickly, and were told that it was the first impressions which were of particular interest. Typically an experimental session lasted between forty-five and sixty minutes. Subjects were tested individually.

RESULTS

No differences were found between the four types of objects placed within the frames, and only combined results will be reported. The method of randomized paired comparisons involves a multiple regression analysis of relative preferences upon horizontal and vertical positions of the object within the frame, and allows the preference function to be extracted as a polynomial to any desired degree of accuracy either for individual or group data (see Appendix for statistical explanation). Fourth-order polynomials appear to be adequate for most purposes, although we describe overall results in terms of sixth-order polynomials.

Figure 5 shows the overall preference functions for all subjects according to the placing of the object within the field, which are statistically highly significant (p < .001). The positions of the golden section divisions of the field are indicated. It is clear that the most preferred horizontal placing is at the golden section, with the mid-point being less preferred, and positions further toward the edge being preferred less still. Preference for vertical placing is far less clear, with a broad plateau, indicting a range of almost complete indifference between the golden sections, and then a rapid falling away of preferences outside of this range. The golden section therefore seems to fulfill different roles: as an optimum in the horizontal domain, and as a set of bounds within the vertical domain.

Figure 6 shows a selection of individual preference functions, which have been chosen to demonstrate the range of functions produced, and each of which is statistically significant (p < .001). It can be seen that in the horizontal domain, there are subjects who prefer objects close to the edge (S.9 and S.21), and who





Figure 5. Preferences for the vertical and horizontal position of an object placed within a rectangular field, shown separately for vertical and horizontal positions. Positions are indicated relative to the total width of the field. The positions of the midline and of a golden section division (φ) are indicated. Functions were calculated separately by 6th order polynomials (solid lines and points), and by 4th order polynomials to indicate that there is little difference due to using the higher order polynomial. For clarity the graphs indicating vertical position have been rotated through a right angle, so that vertical position on the graph maps directly onto vertical position within the frame.



Figure 6. Preference for the vertical and horizontal position within a field shown individually for five subjects chosen for their wide range. Plotting conventions are as in Figure 5. Vertical positions are shown in the top row and horizontal positions in the bottom row.

dislike the mid-line (S.13), and that peaks are not always precisely at the golden section (e.g., S.13 and S.4). Within the vertical domain, none of the subjects showed the plateau of Figure 5, and had instead either single peaks (e.g., S.6), double peaks (e.g., S.4, S.13, and S.21), or even peaks for objects near the bottom of the field (e.g., S.9).

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DISCUSSION

Preference for placing objects within a pictorial field has demonstrated (as did preferences for rectangles and other simple figures), that the golden section manifests principally as a population phenomenon, and that individual preferences reveal much variability, so that the golden section may well not actually be the "most liked" but rather the "least disliked"—the lowest common denominator of a range of different preference functions. This conclusion is not only similar to that reached for rectangles and triangles (McManus,1980), but is also similar to that reached in a study of the aesthetics of color (McManus et al., 1981), in which although there was a clear population preference for blue, a result in accord with many other studies, the overall preference concealed a range of different individual preference functions.

SENSORY, STRUCTURAL AND SCALAR AESTHETICS

Aesthetic phenomena can work at different levels. The simplest is what might be called *sensory aesthetics*. These are the simple pleasures of pure sensations; the taste of strawberries, the blue of the sky, the feel of a book bound in leather, and the sound of a single stroke of a gong. They are what Socrates referred to as "the beauty of figures . . . [produced by] a carpenter's rule and square . . . [which are] beautiful in their very nature" (*Philebus*, 51.c). These phenomena seem so elemental as to be, on first encounter, almost beyond further analysis.

A second type of aesthetic phenomenon is concerned with the interrelationships between components, ignoring the specific content and emphasizing form. Pleasure is directly derived from the analysis and perception of structure, and is gained from dissecting away surface relationships to find the hidden rules that underlie such works as a symphony, a painting, or even a cycle of paintings. The effort after meaning is pleasurable in its own right, and the insights gained provide a deeper understanding and a further liking for the art object. Initially the object may even be disliked if its surface sensory components are not immediately attractive (as for instance with the severely formal works of Webern or Schoenberg), but as structural analysis occurs, so the beauty appears. This process of *structural aesthetics* is particularly manifest in architecture and music (and architecture has been described as "frozen music"), in which formal beauty is particularly important, but is also present even in the most representational of art forms. Economy and elegance of description are also important, and have been well formalized for geometric figures by Boselie and Leeuwenberg (1984) who have applied them with great success to preferences for the well-known geometric figures described by Birkhoff (1933). The structural level of analysis has been implicitly described by George Kelly, in his theory of Personal Constructs (Kelly, 1955), where he argues that human beings are continually trying to produce cognitive models of their world in order to understand and hence to predict it better. More directly, Nick Humphrey (1973; 1983) has argued that aesthetics is of direct evolutionary survival value, precisely because it involves an analysis of underlying relationships, and hence a comprehension of deep, formal relationships. Our pleasure is in part due to being better survival machines.

The third aesthetic phenomenon concerns scale. Some objects are beautiful not merely because of their surface properties or their formal structure but because of their absolute size, or their scale. A small plastic model of the Matterhorn does not evoke the same aesthetic responses as the mountain itself. Neither do the large color field paintings of Mark Rothko evoke a response in small reproductions; and short extracts from Mahler's symphonies do not produce a similar effect to the overwhelming power of two hours of continuous music with its fortissimos and pianissimos. This we can call *scalar aesthetics*. It was first described well by Burke (1757) when he differentiated the "sublime" from the "beautiful."

If the above schematization is correct then it is obvious that the second stage, of structural aesthetics, bears a strong and obvious relation to cognitive theorizing. The act of cognition is an aesthetic act, and aesthetics is a consequence of cognition. Sensory aesthetics seems to be more of a problem: the remainder of this article will be devoted to a hypothesis explaining it also in cognitive terms.

A COGNITIVE THEORY OF SENSORY AESTHETICS

Sensory processes mirror changes in the physical world, and those changes are mainly continuous, an infinitude of small changes being possible between wide extremes. We can produce almost infinite numbers of different colors (and personal computers now routinely advertise a "palette" of 16 million different colors). Similarly a computer screen can display a vast range of quadrilaterals of different size, shape, orientation, and contrast. To understand why there should be aesthetic preferences for certain of this panoply of possibilities is difficult. A solution becomes apparent when we extract individual members of those continua and ask subjects to describe them. Figure 7 shows a range of geometric shapes. Someone shown stimulus A will probably describe it as a "square," and B as a "rectangle"; and might differentiate C and D as "portrait rectangle" and "landscape rectangle," to use the descriptions of dealers in canvasses. E is clearly a "diamond," F a "trapezoid," G a "triangle," and H a "parallelogram" and so on. But what of stimuli like I or J. These are more awkward. They do not fit clearly



Figure 7. A range of quadrilateral and other shapes varying along different continua. See text for details.

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into one category or another. These transitional forms are sitting at the boundaries between one conceptual category and another.

Why do we categorize the continua of sensory stimuli in this way? Presumably in part because it is only possible to have a limited vocabulary (and if we did have an infinite vocabulary for *all* the possible objects that we might seen then when we actually came to communicate about a particular object then either we might not yet have learned its name, or else the person we are communicating with might not know the word we have used). Zipf's law (Zipf, 1949) applies with a vengeance, and the number of tokens must be limited, for communication must involve a reduction in the total amount of information, and must emphasize that which is salient for a particular purpose. Evolutionary pressure therefore means that the tokens which are most useful at differentiating important categorical distinctions in our world will be retained most easily. Or to put it more sensibly, the distinctions which will form first will be those with most pragmatic utility.

This process can be seen most clearly in the particular case of color naming, where the anthropologists Berlin and Kay (Berlin et al., 1969; Kay et al., 1991) have clearly described how three dimensional color space (hue, value, and chroma) is firstly cut into two broad spaces, of "light" and "dark," "black" and "white"; next a space corresponding to the color name "red" differentiates, and this is followed by the space "grue"—a combination of the modern terms for green and blue; and so on. All the evidence is compatible with the idea that this continual splitting of color space occurs in the same evolutionary sequence in all languages. Furthermore this particular ordering of division is clearly related to the neurobiological properties of color sensitive cells in the retina and lateral geniculate nucleus (Kay, 1975; Kay et al., 1978), suggesting that these categories are non-arbitrary, or "natural" (Mervis et al., 1981), being a necessary consequence of the organization of the nervous system (although that nervous system may well have organized itself to respond either to those distinctions which are peculiarly beneficial to survival, or an inevitable characteristic of the way in which the visual world is organized). The ordering of color words described by Berlin and Kay can also be shown to relate closely to the aesthetic use of color words in literature and poetry (McManus, 1983; McManus, 1997).

It should be noted that in Berlin and Kay there is no suggestion that speakers of simpler languages cannot *perceive* or *discriminate* color differences, only that they cannot make adequate generalized verbal distinctions. The color-space may be likened to a map of the Alps, which indicates the height at various places. Initially in trying to describe the space the best description is in broad categories ("Swiss," "Italian," etc.). Eventually we will name every mountain, and there is a sense in which such names are non-arbitrary, reflecting the real topography of the area we seek to describe. After that we may continue to partition, although a certain arbitrariness inevitably creeps in (the distinction, say, between the North and North-West faces of the Eiger, or the pastures of one

farmer from another—the differences are real but are not necessarily related to the physical geography of the region). Of course, one is not suggesting in such a scheme that the mountains cannot be seen; only that they can't be named. But in not being able to name one inevitably limits the social behavior which can be applied to the Alps (Mountaineering Clubs for example would have difficulty in keeping their records). A characteristic of such partitionings is that the earlier ones will be of greater practical use in discriminating any two points chosen at random, and hence will be of more general applicability.

Thus far we have suggested that it is convenient to divide continuous "sensory space" and discontinuous "category space." What happens however when we encounter the transitional forms that are at the boundaries between one category and another? All the evidence suggests that we have difficulty in processing them. Their ambiguity means that reaction times are slower, memory for them is poorer, and so on, at least for color stimuli (Rosch, 1975). How in fact do people describe objects such as I and J in Figure 7? Typically in ways such as "almost a square," or "nearly a triangle, but perhaps a trapezoid." If we wish to manipulate such terms then we cannot use the classical set theory of mathematics and logic, in which an object either is a square or is not a square. Here we are dealing with objects that psychologically both are squares and are not squares at the same time. The appropriate tool is fuzzy set theory (Zadeh et al., 1975; Kosko, 1986; Zwick, 1993), in which an object has many of the properties of a square-more perhaps of a square than of any other category name that we have available-but is still not unambiguously a square. In some circumstances it may sometimes be adequately regarded as a square (a rapid sketch drawing, for instance), whereas in others it most definitely must not be regarded as a square, with all the other properties that that entails (as for instance, if it were the ground plan of a building).

However simple is a stimulus then it must be categorized in some form before we can describe it. Some stimuli we will be able to describe easily because, for cultural, or perhaps biological ("natural"; Mervis et al., 1981), reasons we have a clear category corresponding to them. Others will be close to standard categories and approximate, fuzzy descriptions will be available for them. Many stimuli will however be so transitional that they cannot be adequately described. Nevertheless the attempt at categorization is inevitable and inexorable. Indeed it has been suggested that thought is impossible without it, with Ravenscroft (1996) telling a nice story of how,

In the 1940s, Bertrand Russell was given an IQ test by his friend Rupert Crawshay-Williams. The test involved analogical reasoning about geometrical figures. After making a good start on the simpler problems, Russell found himself incapable of going on, complaining that he "hadn't got any names for the shapes." Unable to *name* the shapes, Russell was apparently unable to *think* about them. [Emphases in the original]

In the same way that we cannot fail continually to make hypotheses about the Necker cube, and hence see it as perpetually reversing in depth, so also we cannot fail to apply our categories to the sensory events with which we are confronted. The effort after meaning and after linguistic categories is eternal. When presented with pairs of rectangles, or triangles, or colored patches, or framed field containing objects, and are asked to make a preference judgment, then we are also categorizing the objects. Individuals may differ in their aesthetic responses for two types of reason. First they may actually have different categories; perhaps when shown a set of isosceles triangles of different proportions, one person sees them as the different types of sails on a boat, whereas another sees them in purely geometric terms. Even if two individuals have the same categories then they may also differ in their response to the classification. A rigorous individual who is intolerant of ambiguity may prefer clear, precise forms at the end of the continuum; and an inquisitive, exploring mind may prefer the marginal forms which are between conventional categories, evoking a pleasing ambiguity. Preferences may also involve the "discrepancy hypothesis," that stimuli are preferred which are slightly, but not too, discrepant from an adaptation level set by repeated exposure.

The present approach clearly is related in part to the controversial suggestion (Martindale, 1984; Martindale et al., 1988) that prototypes are important in aesthetic perception, with both theoretical and empirical arguments being proposed for and against (Boselie,1991; Hekkert et al., 1995; Brant et al., 1995). Like Martindale, the present approach argues for the importance of prototypes, but suggests that they are not the basis of aesthetic judgments *per se*, but are instead the scaffolding upon which aesthetic judgments can be constructed, with individuals showing wide variation in the ways in which they choose to create those judgments. The core problem for aesthetics is to make comprehensible the wide range of individual differences in aesthetic judgments, albeit a range that is not entirely infinite (and hence it is possible to determine population level preferences, albeit small ones).

To encapsulate the theory, all individuals have cognitive categories which divide the continuously variable sensory world into discrete perceptual categories: aesthetic preference functions are the result of fuzzy set manipulations of that categorization, the particular manipulation reflecting the particular categories and the response to ambiguity and novelty of the individual. In so far as some categories, such as colors, are near universal, so population preference functions will be particularly strong. Where categories are more idiosyncratic, then population preferences will be weak in the presence of large individual differences. If some categories are almost inevitable given the constraints upon certain classes of stimuli (such as the division of a line), so that few categories are possible, then population preferences will also tend to be highly organized and similar to one another.

A further extension of the model is that the apparently separate processes of scalar aesthetics may be brought into the same process. The size of objects can be organized into certain natural categories—so that for instance sculpture or pictures may be regarded as "miniature," "hand sized," "half size," "three quarter size," "life size," or "larger than life." These categories can then be compared with actual art objects, and a preference function produced which is a transform of the categories. The idea of categorizing continua into various groups based on scale was suggested by Le Corbusier in his idea of the *Modulor*, and has been further developed by the Dutch architect van der Laan (Padovan, 1986).

What are the implications of this theory of sensory aesthetics for the study of art objects? It is clear that we must differentiate between objects which are grouped by the artist into the same fuzzy set or into different sets. Thus two different greens, both slightly yellowish are not the same as a yellowish green and a greenish yellow which although of equal psychological discriminability as the first pair of colors, are given different names. In exploiting this effect an artist may wish to make unambiguous statements (as does Mondriaan in his later works by using unconfusable primary colors) or alternatively they may wish to emphasize the maximum degree of ambiguity, by using colors within a single fuzzy set, and thereby explicitly describing those very phenomena which cannot be satisfactorily described in words, and provoking a cognitive re-working of the object (as for instance does Vasarely in color in some of his works, or in black and white in his Supernovae, where there is a continual tension as the geometric figures merge and separate as they are classified and re-classified into different categorical perceptions). A more difficult problem for the critic is that they must realize that the categories with which they are viewing a work may be fundamentally different from those of the artist or of its intended viewer, a particular problem with medieval and Renaissance art or particularly with ancient or so-called "primitive art"; in either case one must attempt to view the world through the "fuzzy spectacles" of another classificatory system.

How might one test such a cognitive model? The straightforward way is first to map out the cognitive or category space of the individual within the sensory space which is being assessed, and then map the preference function within the sensory space. If correct then the preference function should be a transform of the cognitive space. An additional complication is that category names, like all names, also exist in "semantic space" (and even simple geometric forms can be analyzed by a semantic differential; Pickford, 1979). Preference judgments may in some circumstances be a joint function of cognitive space and semantic space.

It can now be seen how the theory can explain the data of the current experiment and of that on rectangles. Consider firstly the possible range of rectangular stimuli (of which those in the top row of Figure 7 are a sub set). Figure 8 shows how this continuum may be organized into various categories, and the fuzzy set membership probabilities for stimuli in each of the categories. In effect only

five different categories will be generally available: "Square," "vertical line," "horizontal line," "vertical rectangle," and "horizontal rectangle." The position of the function delineating the two "rectangles" will be a matter for individual variation, but in general it must occur somewhere near the golden section (although there is no reason why it should be precisely at it-the rectangle is not special as such, but rather is highly typical, so that few categories of "rectangle" would not include it somehow). The preference functions of Figure 2 can now be seen to be straightforward transformations of the functions shown in Figure 8. Similarly the field within a frame might be verbally labeled and subdivided using terms such as "left-edge," "right-edge," "left half," "right half," and "middle" for horizontal position; and similar categories for vertical positions (Figure 9); and in an entirely different context, the continuum of a cricket pitch has a wide range of names used to divide up its spatial continuum, all inscrutable to the outsider-"cover," "long off," "silly mid on," etc. Such classification functions can of course be investigated empirically. Some positions would also be classified by their joint position "center," "top left hand corner," which would be intersections of horizontal and vertical sets. Transformations of such functions would produce the sorts of function shown in Figure 6. It should be noted that simply because the English language does not have precise words for these positions, does not mean that they are not conceptual categories, with specific membership functions.

In summary, this article has argued that we can understand the preferences that individuals express during typical experiments on sensory aesthetics as being intimately related to the conceptual categories with which individuals see the world around them. The particular predominance of the golden section does not therefore arise because of its intrinsic mathematical properties, but rather because our minds necessarily classify phenomena, and that classification finds it pragmatically useful for us to have a category called "rectangle," which in general is particularly well satisfied by the actual figure whose ratio is 1:1.6180. But a rectangle of ratio 1:1.55 or 1:1.65 or whatever would almost certainly do as well.



Figure 8. Shows for a continuum of rectangular forms how these might be labelled by various categorical descriptions, for which fuzzy set membership functions, f(..), are indicated.



Figure 9. Shows possible ways in which the vertical and horizontal spaces of a rectangular field might be labelled by means of verbal categories, the bounds indicating the broad range of applicability of each label.

APPENDIX: The Method of Randomized Paired Comparisons

Consider a continuum x, upon which there is a preference function P(x), and that the function can be expressed as a polynomial, so that:

$$P(x) = a + b_1 \cdot x + b_2 \cdot x^2 + b_3 \cdot x^3 \dots$$

Without loss of generalization consider just the cubic polynomial. For two particular positions on the continuum, x_i and x_k , the preferences will be:

$$P(x_j) = a + b_1 x_j + b_2 x_j^2 + b_3 x_j^3$$
$$P(x_k) = a + b_1 x_k + b_2 x_k^2 + b_3 x_k^3$$

The subject expresses a relative preference for x_j versus x_k , resulting in a relative judgment r, where:

$$r = P(x_j) - P(x_k) = b_1 \cdot (x_j - x_k) + b_2 \cdot (x_j^2 - x_k^2) + b_3 \cdot (x_j^3 - x_k^3)$$

Since $(x_j - x_k)$, $(x_j^2 - x_k^2)$, $(x_j^3 - x_k^3)$ are derivable from stimulus properties, then the equation is linear, and estimates of b_1 , b_2 , and b_3 can be derived by conventional least squares fitting using multiple regression, and hence the preference function P(x) can be derived (omitting only the constant term *a*). The method is readily extended to the multidimensional case, where, as in the present case, the stimuli are varying independently on several dimensions such as vertical and horizontal position.

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