

Fractal dimensionality and preference for
synthetic random dot patterns.

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Experimental aesthetics has long tried to use Fechner's method of study "from below", by asking subjects to express preferences for synthetic geometric stimuli, and the method has been successfully applied to objects such as rectangles and triangles (see McManus, 1980). However such stimuli fail conspicuously to encapsulate the complexity and detail either of natural stimuli or of art objects. Attempts to study more complex objects have generally only been concerned with stimuli generated by purely random processes (such as the $N \times N$ black and white matrices generated by Dorfman and McKenna (1966), in which each cell has a 0.5 probability of being black or white). However such stimuli fail as adequate experimental stimuli in that pure mathematical randomness does not seem to be related to the true complexity of natural or artistic objects, even such paintings as the seemingly random Abstract Expressionist paintings of Jackson Pollock revealing themselves as being far more structured and more textured than do such random matrices. In this paper we describe the generation of complex black and white $N \times N$ matrices which are not entirely random, and hence contain structure and organisation, and hence show a greater similarity to natural and art objects.

In recent years there has been a massive surge of interest in fractal geometry (see Mandelbrot, 1977), which is capable of providing useful descriptions of such complex natural objects as coastlines, clouds and mountains, which have defied conventional analysis. Essentially fractal geometry considers objects such as coast-lines to have dimensions which are intermediate between the integers required by Euclidean topology. The line of a coast is therefore of higher dimension than a straight line (of dimension 1), but of lesser dimension than an area (of dimension 2), the fractal dimension becoming higher and higher as the line fills more and more of space, until eventually in such complex objects as the Peano curve the line fills two-dimensional space and achieves a dimension of 2. The important feature for present purposes is that dimensionality is a measure of complexity, objects of lesser dimensionality being more auto-correlated.

Voss and Clarke (1975) have shown that such fractal dimensions are a good description of the temporal patterns found in speech and music, and

other studies (Gardner, 1978) have synthesised music of fractional dimension between 1 and 2, and shown that music is preferred if it has an intermediate dimensionality ($1/f$) between that of pure randomness ("white noise") and the very slowly changing structure of Brownian motion ("brown noise").

In this paper we describe how we have used a method of Fourier synthesis (Barnsley *et al*, 1988, p 90 *et seq*) to create black-white matrices that topologically are two-dimensional but have fractional geometrical dimensions of varying degrees.

Method.

Subjects. 43 subjects (16 male, 27 female) took part in the experiment, most of whom were undergraduates at University College London. Ages ranged from 17 to 65 (mean = 22.7 years).

Stimuli. On each trial subjects were presented with a pair of $N \times N$ matrices which differed in their fractal dimension. Exactly 50% of the cells in each matrix were black and 50% were white. A Fourier Synthesis method was used to create the stimuli in which an appropriate amplitude and phase spectrum were created, and then an Inverse Fast Fourier Transform was used to create the image corresponding to the spectrum. Specifically the amplitude spectrum was created such that,

$$\log(\text{power}) = -k \cdot \log(\text{frequency})$$

where k is related to the fractal dimension, such that $k=0$ ($1/f^0$) correspond to a completely random stimulus of white noise, and higher values, of $k=1, 2$, and 4 ($1/f^1$, $1/f^2$, $1/f^3$ and $1/f^4$) produce progressively more auto-correlated stimuli. Each element of the phase spectrum was set to a random value between $-\pi$ and $+\pi$, with the single exception that the phases of the fundamental components were set at 0, so that the image would appear as a central white area against a black background. Figure 1 shows examples of stimulus pairs of different values of k and different sizes. Computations were carried out on a VAX11/780 computer, and stimuli were generated on a Sperry 37 laser printer.

Procedure. Each subject saw stimuli of size 8×8 , 16×16 , 32×32 and 64×64 , and within each size saw all possible pairs of fractal dimensions corresponding to $1/f^0$, $1/f^1$, $1/f^2$, $1/f^3$ and $1/f^4$. The design was therefore of a complete paired comparisons within sizes, with each fractal dimension occurring equally often to right and left. The subjects therefore saw a total of 8 pairs of stimuli. Preference judgements were made on a six-point scale which was reduced to a binary choice (left or right) before analysis.

Results.

For each subject and each stimulus size the set of all possible paired comparisons across fractal dimensions was analysed to obtain a single score giving the preference for each fractal dimension relative to the other fractal dimensions; a score of +1 indicated that it was preferred to all other dimensions and a score of -1 indicated that it was disliked relative to all

other dimensions. The preference scores for each dimension were then subjected to a repeated measures analysis of variance.

Figure 2 shows the preference for each fractal dimension (expressed as the slope) for each size of stimulus. Analysis of variance shows highly significant effects of slope ($F(4,160)=5.18$, $p<.001$), which was due to a significant linear component ($F(1,160)=18.49$, $p<.001$), and no evidence of a quadratic or higher order trend. There was also a significant interaction between slope and stimulus size ($F(12,480)=6.25$, $p<.001$), which was almost entirely due to a linear \times linear component ($F(1,480)=66.18$, $p<.001$). Because of the nature of the design it was not possible to compare preference for different matrix sizes.

Discussion.

We have shown that synthetic stimuli can be produced which are suitable for studies in experimental aesthetics, having the richness and structure characteristic of natural and artistic objects, but being sufficiently controlled to allow experimental manipulation.

Subjects show clear preferences for stimuli which differ in their fractal dimensionality. However in comparison with the work described by Gardner (1979) we could find no evidence for an optimal amount of complexity; preference increased linearly from the $1/f^0$ to $1/f^4$ stimuli. It is possible that the population results described here are hiding significant individual differences in preference (as has been described for simple geometric figures; McManus, 1980). That possibility is being investigated at present.

References.

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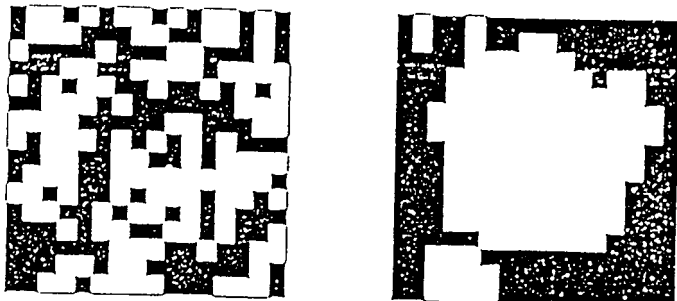
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Figure 1. Shows two pairs of stimuli used in the study. a). Stimuli of size 16x16 with slopes of $1/f^0$ (left) and $1/f^2$ (right); b). Stimuli of size 64x64 with slopes of $1/f^0$ (left) and $1/f^2$ (right).

a).



b).

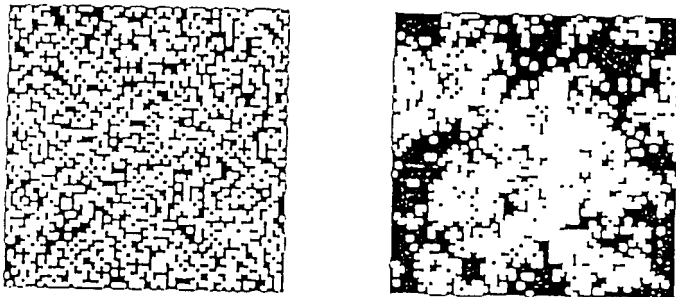


Figure 2. Shows the overall preferences of subjects for the stimuli of different fractal dimensions (slope) for each separate size of stimulus.

