Second year projetcs (Jean-Marc Vanden-Broeck)

The proposed projects are concerned with the theory of linear and nonlinear water waves. The suggested references are based on my book 'Gravity-capillary free surface flows, Cambridge University Press 2010' which is avaiable in electronic format in the UCL library. The numbers in squared brackets refer to the papers listed in the bibliography. The book is referred as [A].

GRAVITY WAVES

Consider a wave propagating at a constant velocity U at the surface of an incompressible and inviscid fluid of finite depth H. Assume that the flow is irrotational and take a frame of reference moving with the waves. Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the gravity g but neglect surface tension. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations by separation of variables. Find the dispersion relation. Determine for which values of the Froude number $F = U/(gH)^{1/2}$, solutions exist. Introduce the concept of group velocity and explain why waves occur at the back of a disturbance moving at a constant velocity U.

REFERENCES: [A] Section 2.4

GRAVITY-CAPILLARY WAVES

Consider a wave propagating at a constant velocity U at the surface of an incompressible and inviscid fluid of finite depth H. Assume that the flow is irrotational and take a frame of reference moving with the waves. Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the gravity g and the surface tension T. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations by separation of variables. Write the dispersion relation in the form of an equation relating the Froude number $F = U/(gH)^{1/2}$, the Bond number $\tau = T/(\rho g H^2)$ and the dimensionless number $l = \lambda/H$. Here ρ is the density of the fluid and λ is the wavelength. Plot F as a function of l for various values of τ and show that there is a minimum when $\tau < 1/3$. Introduce the concept of group velocity. Assume now $\tau < 1/3$. Explain why long waves occur at the back of a disturbance moving at a constant velocity U while short waves occur at the front.

REFERENCES: [A] Section 2.4

CAPILLARY WAVES

Consider a wave propagating at a constant velocity U at the surface of an incompressible and inviscid fluid of infinite depth. Assume that the flow is irrotational and take a frame of reference moving with the waves. Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the surface tension T but neglect gravity. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations by separation of variables. Find the dispersion relation. Show that the fully nonlinear problem has an exact solution (see [37]). Use this exact solution to plot wave profiles and streamlines. Discuss the limiting configuration.

References: [A] Section 6.5.1 and [37]

LIMITING CONFIGURATION OF GRAVITY WAVES

Consider a wave propagating at a constant velocity U at the surface of an incompressible and inviscid fluid of infinite depth. Assume that the flow is irrotational and take a frame of reference moving with the waves. Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the gravity g but neglect surface tension. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations by separation of variables. Find the dispersion relation. Consider now the fully nonlinear problem. Show that the limiting configuration is characterised by an angle at the crest with an emclosed angle of 120 degrees. Describe a method to compute this limiting configuration and as time permits calculate it.

References [A]: Section 3.3 and [172]

KORTEWEG DE VRIES EQUATION: GRAVITY

Consider a wave propagating at a constant velocity U at the surface of an incompressible and inviscid fluid of finite depth H. Assume that the flow is irrotational and take a frame of reference moving with the waves. Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the gravity g but neglect surface tension. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations by separation of variables. Find the dispersion relation. Expand the dispersion relation in powers of the wavenumber k. Deduce from this expansion the linearised form of the Korteweg de Vries equation. Consider now the complete weakly nonlinear Korteweg de Vries equation. Find periodic solutions (the so called cnoidal waves). Plot these solutions for various values of the parameters. Show that the cnoidal waves approach solitary waves as the wavelength tends to infinity.

References [A] Section 5.2 and [195]

KORTEWEG DE VRIES EQUATION: GRAVITY AND SURFACE TENSION

Consider a wave propagating at a constant velocity U at the surface of an incompressible and inviscid fluid of finite depth H. Assume that the flow is irrotational and take a frame of reference moving with the waves. Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the gravity g and the surface tension T. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations by separation of variables. Find the dispersion relation. Expand the dispersion relation in powers of the wavenumber k. Deduce from this expansion the linearised form of the Korteweg de Vries equation. Consider now the complete weakly nonlinear Korteweg de Vries equation. Find periodic solutions (the so called cnoidal waves). Plot these solutions for various values of the parameters. Show that the cnoidal waves approach solitary waves as the wavelength tends to infinity. Show that the solitary waves are elevation waves for some values of the Bond number $\tau = T/(\rho g H^2)$ and depression waves for others. Determine these values. What happens as $\tau \to 1/3$?

References [A] Section 5.2 and [195]

FIFTH ORDER KORTEWEG DE VRIES EQUATION: GRAVITY AND SURFACE TENSION

Consider a wave propagating at a constant velocity U at the surface of an incompressible and inviscid fluid of finite depth H. Assume that the flow is irrotational and take a frame of reference moving with the waves. Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the gravity g and the surface tension T. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations by separation of variables. Find the dispersion relation. Expand the dispersion relation in powers of the wavenumber k. Deduce from this expansion the linearised form of the fifth order Korteweg de Vries equation. Describe the Wilton ripples.

References [A] Section 5.2 and [76]

WAVES GENERATED BY A MOVING PRESSURE DISTRIBUTION: GRAVITY, INFINITE DEPTH

This project is concerned with the free surface flow generated by a distribution of pressure moving at a constant velocity U at the surface of an inviscid and incompressible fluid of infinite depth. Assume that the flow is irrotational and take a frame of reference moving with the pressure distribution.

Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the gravity g but neglect surface tension. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations. Discuss the solutions. You might have to introduce an artificial viscosity. Find the amplitude of the waves in the far field.

References [A] Chapter 4

WAVES GENERATED BY A MOVING PRESSURE DISTRIBUTION: GRAVITY, FINITE DEPTH

This project is concerned with the free surface flow generated by a distribution of pressure moving at a constant velocity U at the surface of an inviscid and incompressible fluid of finite depth. Assume that the flow is irrotational and take a frame of reference moving with the pressure distribution. Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the gravity g but neglect surface tension. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations. Discuss the solutions. You might have to introduce an artificial viscosity. Compute the solutions when the Froude number is bigger than 1 and display the free surface profiles.

References [A] Chapter 4

WAVES GENERATED BY A MOVING PRESSURE DISTRIBUTION: GRAVITY, SURFACE TENSION, INFINITE DEPTH This project is concerned with the free surface flow generated by a distribution of pressure moving at a constant velocity U at the surface of an inviscid and incompressible fluid of infinite depth. Assume that the flow is irrotational and take a frame of reference moving with the pressure distribution. Formulate the fully nonlinear problem in terms of a potential function $\phi(x, y)$. Include the gravity g and the surface tension. T. Derive a linear theory by assuming a small disturbance around a uniform stream with constant velocity U. Solve the resulting equations. Discuss the solutions. You might have to introduce an artificial viscosity.

References [A] Chapter 4