MATH0066 (Nonlinear Systems)

Year:	2024–2025
Code:	MATH0066
Level:	Masters
Value:	15 UCL credits (= 7.5 ECTS credits)
Term:	1
Structure:	3 hours lectures per week
Assessment:	100% examination. Weekly Homework Problem Sheets for practice (marked and returned to provide feedback).
Lecturer:	Dr Steve Baigent

Course Description and Objectives

This module gives an overview of the main aspects of nonlinear systems that arise in continuous and discrete dynamical systems and aims to build the required skills to understand and visualise various dynamical outcomes. It will provide basic definitions and indicate theoretical background. Physical applications will be considered with an emphasis placed on combining analytical treatments with an understanding of their practical implications in a modelling setting.

Recommended Texts

(i) S.H.Strogatz, *Nonlinear Dynamics and Chaos*, Perseus Books, 1994.

(ii) J.M.T.Thompson & H.B. Stewart, *Nonlinear Dynamics and Chaos*, Wiley 2003.

(iii) E.Ott, *Chaos in Dynamical Systems*, CUP, 1993.

(iv) D.K.Arrowsmith & C.M.Place, *Dynamical Systems*, Chapman Hall 1990.

(v) J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields*, Springer 1983.

(vi) P.Drazin *Nonlinear Systems*, Cambridge University Press, 1992.

(vii) P.Drazin and R.S.Johnson, *Solitons*, Cambridge University Press, 1989.

(viii) L.D.Landau & E.M.Lifshitz, *A Course of Theoretical Physics*, Vol. 1 Mechanics, Pergamon 1960.

(ix) E.Infeld & G.Rowlands, *Nonlinear Waves, Solitons and Chaos*, CUP, 2000.

(x) L.N.Virgin, *Introduction to Experimental Nonlinear Dynamics*, CUP, 2000.

Detailed Syllabus

Introduction: Review of dynamics and modelling concepts. Deterministic versus probabilistic models. The role of nonlinearity and damping/dissipation. Introduction to complex systems and agent based modelling.

Continuous Dynamical Systems: Flows governed by ODEs. Transients and steady states - equilibrium, periodic and chaotic solutions. Local and global stability. Liouville's theorem. Conservative and dissipative mechanical systems. Periodic solutions and Poincare-Bendixson theorem. Bifurcation theory for 1- and 2-dimensional systems including structural stability of bifurcations. Comparison of conservative systems and dissipative systems. Chaos and the butterfly effect. Potential well dynamics for nonlinear oscillators. Numerical considerations including basins of attraction, the role of unstable saddles, homoclinic/heteroclinic trajectories and Lyapunov exponents. Link to maps via Poincaré sections.

Discrete Dynamical Systems: Iterated maps as dynamical systems in discrete time. The logistic map as main example. Fixed points, cycles and their stability. Period doubling and other bifurcations of maps. Elementary properties of maps in two dimensions. Lyapunov exponents for maps.

Spatial Temporal Dynamics: Flows governed by PDEs. Linear waves, dispersion relations, dissipation leading to stable waves. Travelling wave solutions of non-linear partial differential equations, for example the Korteweg-de Vries. Solitons and link with homoclinic orbits of potential well systems.

August 2024