Project Titles for MATH0084
2024-25

Suggested projects for 2024-2025 are arranged here by broad subject area and alphabetically by supervisor. Most projects have either prerequisite or suggested modules. **Please contact supervisors directly** if you would like to find out more information about a project or to register an interest in taking that project.

Contents

1 Mathematical Biology and modelling

- **Dr Stephen Baigent** steve.baigent@ucl.ac.uk

  1. *Optimal habitat choice with travel costs*

     This project builds on one supervised 2 years ago. Imagine a large population consisting of several species that populate a fixed habitat. Each habitat has a range of resources and each species has food preferences, safety concerns, etc. How does the population fill out the habitat? This is an evolutionary game and the choice is usually a special kind of Nash equilibrium known as an evolutionarily stable state. If it actually costs individuals to move between sites, this complicates the problem: They may not move even if they would be 'fitter' in the new site if it costs too much in fitness terms to reach it. The theory for this is not so well known, and possibly not known for some models. The aim will be to formulate a new 2D partial differential equation model where there is not a finite number, but a continuum of species. The first task will be to set up the model and show that it makes sense (has meaningful solutions for reasonable scenarios) and the second task will be compute these solutions using finite-differences, finite elements, or similar, using Mathematica, Python, Matlab or a suitable pde solver, and explore cost-benefit trade-offs for different models.

Pre-requisites: Some experience of programming would be useful. While the project includes some game theory ideas covered in the 2nd term module Evolutionary Games and Population Genetics (MATH0082), the project can be done independently.

2. *Invariant manifolds of discrete-time dynamical systems*

   Many models that arise in theoretical ecology and evolutionary game theory have curves, surfaces, or more generally manifolds, that are left invariant by the dynamics. For models that have an invariant manifold that attracts all points, the model can be solved by restricting to the manifold, generally an easier problem. The first aim of the project will be to learn some key mathematical theory for showing when these invariant manifolds exist, and then to apply the theory to several well-known models from theoretical ecology. A second aim will be to find (hopefully new) ways of computing these invariant manifolds using a program such as Mathematica, Python or Matlab, and then use the computations to push the models to limits where the invariant manifolds lose smoothness and eventually disappear.

   Pre-requisites: Some experience of programming would be useful. No Mathematical Biology background is needed.

- **Dr Freya Bull** f.bull@ucl.ac.uk

1. *Bacterial growth in complex environments* In nature, bacteria frequently grow in complex environmental conditions. In applications including pharmaceutical production, food/drink manufacturing, marine ecology, and bacterial infections within the gut/bladder; bacteria encounter low concentrations of many diverse nutrients. This project will investigate how bacteria uptake multiple substrates (nutrients required for growth) simultaneously: an important missing piece in modelling bacterial growth outside of the laboratory. Potential directions a student could take this project in include (but are not limited to): (a) developing and computationally implementing an agent-based stochastic model; (b) developing and numerically solving continuum PDE models; or (c) developing (mathematical) continuum models for multi-substrate growth modes and evaluating these models as (higher order) corrective terms to a single-substrate growth model.
2. *Bacterial ascension of catheters* Urinary catheters – thin tubes used to drain the bladder – are commonly used, both in hospitals and in long-term care facilities. Unfortunately, urinary catheters are prone to colonisation by bacteria, leading to the development of urinary tract infections. Recent work has suggested the time taken by the bacteria to ascend the catheter plays a key role in determining the incidence of these infections. The primary aim of this project is to construct a mathematical model of bacterial ascension of a catheter by considering three compartments: the catheter surface, the uroepithelial cells, and a mucosal layer. A secondary aim would then be to apply this model to determine: (a) how the bacterial & host characteristics determine the timescale of ascension, and (b) what conditions prevent the successful ascent of bacteria (e.g. considering the host immune response, and/or antimicrobial treatment).

Prerequisites: Comfort with differential equations and some programming experience. No required modules, but useful modules would be Mathematical Ecology, Mathematical Methods 4, Mathematical Methods 5, and Advanced Modelling Mathematical Techniques.

- **Dr Rosemary Harris** rosemary.j.harris@ucl.ac.uk

  1. *Record statistics and applications*

     In a time series of data (e.g., daily stock prices, yearly mean temperatures) we can identify a record value when an element of the series is higher (or lower) than all previous elements. How is the frequency of such records expected to depend on time? By observing records can we say anything about the time-dependence of the underlying process (e.g., financial trends, climate change)? This project will investigate these questions starting with the theory of record values for simple random walk models. The student will be expected to reproduce some results, numerically and/or analytically from a paper by Wergen et al. [Phys. Rev. E 83 051109 (2011)], test the claims there on a different dataset and possibly consider further extensions.

  2. *Modelling distorted memory*

     The "peak-end rule" of Kahneman et al. [Psychol. Sci. 4 (1993) 401–405 (1993)] is a psychological heuristic reflecting the fact that human recall of past experiences is dominated by extreme events on the one hand, and recent events on the other. This distorted memory can affect our future decisions in interesting ways as modelled recently by Mitsokapas and Harris [Physica A 593, 126762 (2022)]. The aim of this project is to understand and reproduce some of the calculations in that work and in particular to consider numerically and/or analytically how the process can be optimized when the "noise" in the decision-making depends on time. For an ambitious student, the model could then be generalized in various ways or even perhaps compared with real data from online experiments.

  Prerequisites for both: Some familiarity with stochastic processes and programming experience (in any language); growing confidence in the topics of MATH0065 Advanced Modelling Mathematical Techniques (ideal co-requisite)

- **Prof Nick Ovenden** n.ovenden@ucl.ac.uk

  1. *Ultrasound contrast agents*

     Microbubbles and phase change contrast agents have many potential clinical applications in terms of ultrasound imaging, embolic occlusion therapy, drug delivery and high intensity focussed ultrasound. Their uptake in clinical practice, however, requires much better understanding of the behaviour of suspensions in vivo. The mechanics of tiny bubbles and droplets in an ultrasound field can be modelling via nonlinear systems of ODEs and PDEs. Possible projects include (i) modelling the vaporization process of nanodroplets, (ii) exploring bubble-nanodroplet interactions (including coalescence) or (iii) surfactant shedding of bubble coatings.

     Recommended pre/co-requisites: MATH0027 (Methods 5), Real Fluids MATH0078 (Asymptotic methods and Boundary layer theory), MATH0080 (waves and wave scattering). Some experience with programming in Python/Julia or MATLAB is useful.

  2. *Physiological modelling of critically-ill patients*

     This project will involve working in the mathematics in healthcare hub CHIMERA at UCL looking at biomechanical models of critically-ill patients in intensive care. The project is likely to involve collaboration with clinician. Systems of equations to replicate the respiratory and/or cardiovascular systems will be explored and validated against real-patient data. Examples of projects include airway secretions and clearance or modelling gas exchange in the lung during mechanical ventilation. The project may also incorporate data science/machine learning techniques.

Recommend pre/co-requisites: MATH0027 (Methods 5), Real Fluids (MATH0077) and some knowledge of computational methods for differential equations (MATH0033, MATH0058) is desirable. Some experience with programming in Python/Julia or MATLAB is useful.

- **Prof Karen Page** karen.page@ucl.ac.uk

  1. *Mathematical models of diatom ecology*

     Diatoms are phytoplankton with beautiful glassy shells (see https://diatoms.org/what-are-diatoms for more details). The student will review species interactions between diatom species and their relevant predators (e.g. herbivorous copepods [1]), and study diatom spatial distributions and movement. They will build ecological models of a selected species of diatom, studying spatial distributions and species interactions. They may also apply species diversity measures.

     [1] Pohnert, G., 2005. Diatom/copepod interactions in plankton: the indirect chemical defense of unicellular algae. ChemBioChem, 6(6), pp.946-959.

     Prerequisites: MATH0030, programming experience, differential equations. Knowledge of fluid mechanics, especially MATH0024, is also advantageous.

  2. *Information Theory of Chemotaxis*

     The movement of cells in response to chemical gradients (chemotaxis) is important in embryonic development and the immune system, among other processes. Cells sense tiny spatial differences in concentration across a very large range of average concentration. In this project the student will use methods from information theory to assess the limits of detection of these gradients and how cells perform so well.

     Pre-requisites: programming experience, knowledge of differential equations.

- **Dr Philip Pearce** philip.pearce@ucl.ac.uk

  1. *Modelling blood flow in vascular networks*

     An intricate network of vessels transports blood between the heart and the rest of the organs in the human body. This project will begin with a review of theoretical models for blood flow in single idealised tubes and in networks of small blood vessels called capillaries. The aim will be to write code in e.g. Python or Matlab to simulate blood flow in various network topologies, and if possible to test how different assumptions about blood properties can be incorporated into such models.

     Pre-requisites: some programming experience; Real Fluids (a co-requisite).

  2. *Multi-scale modelling of living matter*

     The properties and dynamics of biological tissues, organisms and populations emerge from physical and chemical interactions at the levels of molecules and cells. This project can focus on any of these length scales, and can involve a computational or analytical approach. Example projects include: simulating interactions between extracellular matrix proteins in bacterial biofilms; simulating cell populations at the cellular level; or modelling bacterial populations or tissues using a continuum approach.

Pre-requisites: Some programming experience; Mathematical Methods 4

- **Dr Benjamin Walker** benjamin.walker@ucl.ac.uk

  1. *Models of tumour spheroid growth* Mathematical models of tumour growth have been around for decades, leading to a range of different modelling approaches. This project will consider simple models of tumour development and compare them, exploring how the choice of model can impact the conclusions that we might draw. In particular, the project will look at how the effects of treatment can be incorporated into the various models and whether we can meaningfully translate parameters between them. The final goal will be to (numerically) assess how important the choice of model is in optimising dosage scheduling, a key question in modern healthcare.

     Prerequisites: Familiarity with MATH0030 (Mathematical Ecology) and non-linear ODEs (as taught in MATH0027) is desirable. Experience with programming in Python or MATLAB is helpful but necessary.

  2. *Measuring microswimming* Small-scale swimmers often move by beating a long, slender tail in a sinusoid-like shape. However, the details of this beat, such as its amplitude or frequency, may vary over time. This project will explore and test new ways to define, measure, and analyse the properties of the beating tail as they change over time. This will be tested against synthetic data and then applied to the results of real-world imaging from a canonical microswimmer. Despite the biological application, no familiarity with fluid mechanics is required.

     Prerequisites: Experience with programming in Python or MATLAB is recommended.

  3. *Asymptotics of oscillatory swimming* Many swimmers oscillate rapidly as they swim, leading to trajectories that look smooth over long timescales but are intricate and complex over short timescales. This project will combine a multiple-scales asymptotic analysis of simple ODE models with numerical simulation, looking to determine the relationship between small-scale oscillations and large-scale behaviours of self-propelled particles, potentially including the effects of fluid flow.

     Prerequisites: Some familiarity with asymptotics, such as MATH0078 and the method of multiple scales, is needed. Experience with programming in Python or MATLAB is helpful but necessary.

- **Prof Alexey Zaikin** alexey.zaikin@ucl.ac.uk

  1. *Modelling complex behaviour of organ-on-a-chip platforms with linked mechanosensitive and genetic dynamics*

     Organ-on-a-chip platforms have the potential to accurately predict human physiology and, especially, diseases. The idea of the project is to develop for the first time a mathematical model of a such a system with linked mechanosensitive viscoelastic properties and complex dynamics of intracellular genetic networks. Recently, we have shown that genetic networks may have very complex dynamics [1-3]. On the other hand, recent studies suggest bidirectional causal links between cellular clocks and mechanotransduction [4]. The modelling can be interesting for investigation of organ-on-a-chip systems, which aim to mimic and predict organ-level human physiology by incorporating 3D co-culture of multiple cell types and physiologically relevant mechanical stimuli to recapitulate the in vivo cellular environment [5].

The project is of a computational nature and will require numerical simulations and solutions of a system of coupled ordinary differential equations.

[1]. L. Abrego, and A. Zaikin, "Integrated Information as a Measure of Cognitive Processes in Coupled Genetic Repressilators", Entropy 21(4), 382 (2019). [2]. R. Bates, O. Blyuss, and A. Zaikin," Stochastic resonance in an intracellular genetic perceptron", Phys. Rev. E, 89, 032716 (2014). [3]. Y. Borg, E. Ullner, A. Alagha, A. Alsaedi, D. Nesbeth, and A. Zaikin, "Complex and Unexpected dynamics in Simple Genetic Regulatory Networks", IJMPB 28, 1430006 (2014). [4]. Yang, N et al. Nat Commun (2017). DOI: 10.1038/ncomms14287 [5]. Thompson, CL et al. Front Bioeng Biotechnol (2020).

2 Fluids and modelling

- **Dr Mohit Dalwadi** m.dalwadi@ucl.ac.uk

  1. *Fundamental models of multiscale mass and fluid transport*

     Multiscale problems of mass transport are ubiquitous in physical applied mathematics. Applications include fluid transport in tumours, membrane filtration, nutrient delivery to plant roots in soil, salt transport in sea ice formation, and many more. In this project, the student will review basic partial differential equation models for multiscale mass and fluid transport, and go on to investigate asymptotic solution structures when regions involving different dominant transport mechanisms are coupled together. There are also opportunities - but no requirements - to write numerical simulations in this project.

     Pre-requisites: Advanced Modelling Mathematical Techniques (co-requisite), Asymptotic Approximation Methods (co-requisite), Mathematical Methods 5 [or a willingness to learn aspects of each].

  2. *Cryopreservation*

     Cryopreservation technology is used for applications involving fertility, tissue transplantation, and the protection of endangered species. Mathematical models can be used to understand how to reduce cell damage in this process. Since freezing and melting involve transitions between ice and water phases, mathematical models of this process can involve solving partial differential equations with moving boundaries, where the position of the domain boundary must be determined as part of the solution. In this project, the student will review basic mathematical models for freezing, then investigate how adding cryoprotective chemicals can reduce cell damage in cryopreservation. There will be opportunities to use both asymptotic and numerical methods in this project.

     Pre-requisites: Advanced Modelling Mathematical Techniques (co-requisite), Asymptotic Approximation Methods (co-requisite), Mathematical Methods 5 [or a willingness to learn aspects of each].

  3. *Decontaminating chemical agents*

     When harmful chemical agents are spilled it can be incredibly harmful to people and the environment, so it is vital to be able thoroughly decontaminate affected areas. Mathematical models can be used to understand how to choose appropriate cleansers when confronted with novel chemical agents in the field. Such models typically involve solving partial differential equations with moving boundaries, where the position of the domain boundary must be determined as part of the solution. In this project, the student will review basic mathematical models for chemical decontamination, then explore more complex set-ups, such as emulsions of agent and cleanser. There will be opportunities to use both asymptotic and numerical methods in this project.

Pre-requisites: Advanced Modelling Mathematical Techniques (co-requisite), Asymptotic Approximation Methods (co-requisite), Mathematical Methods 5 [or a willingness to learn aspects of each].

- **Prof Gavin Esler** j.g.esler@ucl.ac.uk

  1. *Floquet analysis of the quasi-biennial oscillation*

     The quasi-biennial oscillation (QBO) is an approximately 28 month oscillation in the east-west winds observed in the equatorial stratosphere (15-35 km above the Earth's surface). The physical mechanism for the oscillation is quite remarkable: waves generated much lower in the atmosphere propagate upwards and as they do so they transport and deposit momentum into eastward and westward jets. Amazingly this physics can be captured in a single equation: the Holton-Lindzen-Plumb equation for the wind U(z,t). The Holton-Lindzen-Plumb equation supports periodic QBO-like solutions, and the project will study the stability of these solutions using a branch of mathematics known as Floquet analysis. The project is suitable for applied-minded students, who enjoy waves, fluid dynamics and stability problems (although not much prior knowledge is needed), and who don't mind doing some supporting numerical calculations.

  2. *Stochastic differential equation models for particle pairs in turbulence*

     There exists an excellent stochastic differential equation (SDE) model (the Langevin equation) for the motion of a single fluid particle in three-dimensional isotropic homogeneous turbulent flow. The situation is much less clear, however, when it comes to a stochastic model for the separation and relative velocity of a pair of particles. The many model equations in the literature all suffer from the same defect - they are essentially inconsistent with the single particle Langevin model.

     The aim of this project is to review some of the particle pair SDE models which appear in the literature and compare their properties with those of a new model. A key element of the project is understanding what the ideal properties a particle pair SDE model should posses. Both analytical and numerical directions are possible, including asymptotic analysis of the Fokker-Planck equation and cross-validation with numerical integrations of SDEs.

     The project is suitable for students who like applied stochastic differential equations and (possibly) asymptotic analysis.

- **Dr Luca Grieco** l.grieco@ucl.ac.uk

  1. *Optimising lecture timetabling at UCL*

     Timetabling of lectures is currently conducted semi-automatically at UCL. Departments raise their requests for specific rooms and time slots which are recorded into a system that detects conflicts. Subsequently, the Timetabling Team try to resolve conflicts by negotiating with the involved departments and other potential rooms users. In this project, the student will explore scientific literature and discuss with the Timetabling Team to formalise the problem of lecture timetabling (or a specific aspect of it), develop its mathematical programming formulation, and analyse its solutions in different scenarios possibly based on suggestions and requirements from the Timetabling Team.

- **Prof Ted Johnson** e.johnson@ucl.ac.uk

  1. *Laplace's equation in domains with corners*

     This project considers a method for computing potential flows in planar domains put forward by Peter Baddoo. The approach is based on a new class of techniques, known as "lightning solvers", which exploit rational function approximation theory in order to achieve excellent convergence rates. The method is particularly suitable for flows in domains with corners where traditional numerical methods fail. The project will outline the mathematical basis for the method and establish the connection with potential flow theory. In particular, the new solver will be applied to a range of classical problems including steady potential flows, vortex dynamics, and free-streamline flows. The solution method is extremely rapid and usually takes just a fraction of a second to converge to a high degree of accuracy. Numerical evaluations of the solutions can be performed in a matter of microseconds and can be compressed further with novel algorithms. The method is described in the paper *'Lightning Solvers for Potential Flows', Peter J. Baddoo, Fluids 2020, 5, 227; doi:10.3390/fluids5040227*

     Prerequisites: Complex Analysis, Matlab or Python skills

  2. *A terminating vortex sheet*

     There exists a simple solution for a steady vortex sheet terminating at a wall. However it is very likely that this steady solution is unstable i.e. that a small perturbation will grow arbitrarily large. This project aims to consider the linear stability problem and will involve solving finding the eigenvalues of a matrix using Matlab.

  3. *Gyres on a beta-plane*

     When long lived eddies form in the oceans the potential vorticity within the eddies tends to become uniform over time. This project will consider simple numerical techniques for describing some of these situations. A knowledge of simple Matlab programming will be needed.

- **Dr Catherine Kamal** ck620@cam.ac.uk

  1. *Modelling the Flow of 2D Materials*

     2D materials, such as graphene and carbon nanotubes, are ubiquitous, used in everything from drug delivery to electronics. Made from just a few atomic layers, visualisation of the instantaneous dynamics of these colloidal particles in flowing liquids is practically, experimentally, inaccessible. The aim of this project is to review colloidal theories to predict the rotational dynamics of slender particles, similar to graphene and carbon nanotubes, in flow. The student will then have the opportunity to run computational simulations to analyse the effect of thermal fluctuations, which arise from the random motion of the fluid particles, on the rotational dynamics of a slender particle. The option to do analytical theory for the particle dynamics will also be available. In doing so, the project's goal is to make measurable predictions on the flow dynamics of 2D materials which are to be compared to Molecular Dynamics simulations (see project "The Flow of Carbon Nanotubes").

Prerequisites: Fluid Mechanics (MATH0015) and Mathematical Methods 3 (MATH0016) are essential. Computational Method (MATH0058) and experience with MATLAB are desirable but not essential.

2. *The Flow of Carbon Nanotubes*

   Carbon nanotubes are ubiquitous, used in everything from inkjet printing of flexible electronic tracks to the design of more robust composites. Made from just a few atomic layers, visualisation of the instantaneous dynamics of these colloidal particles in flowing liquids is practically, experimentally, inaccessible. The aim of this project is to review colloidal theories to predict the rotational dynamics of slender rod-like particles, similar to carbon nanotubes, in flow. The student will have the opportunity to analyse Molecular Dynamics simulations of a sheared carbon nanotube in water. The option to run their own Molecular Dynamics simulations will also be available, although not required. In doing so, the project's goal is to analyse the flow dynamics of a carbon nanotube which is to be compared to continuum theory (see project "Modelling the Flow of 2D Materials").

   Prerequisites: Fluid Mechanics (MATH0015) and Mathematical Methods 3 (MATH0016) are essential. Computational Method (MATH0058) is desirable but not essential.

- **Prof Robb McDonald** n.r.mcdonald@ucl.ac.uk

1. *Interface growth in two dimensions*

   The nonlinear dynamics when an interface deforms in response to a quantity diffusing toward it generates remarkable patterns e.g. viscous fingering, branching stream networks and fractal-like structures formed in electro-deposition. This project will use complex analysis and simple numerical models to explore related models, such as Loewner growth, diffusion-limited aggregation, needle models, and the connections between them.

   Pre-requisites: Fluid mechanics (MATH0015) and Complex analysis (MATH0013). Willingness to use and adapt existing numerical models.

2. *Vortex dynamics*

   Investigate and develop analytical and numerical constructions of equilibria for the 2D Euler equations having non-zero vorticity distributions in the form of points, sheets and patches.

   Pre-requisites: Knowledge and enthusiasm for Fluid mechanics (MATH0015 and Complex analysis (MATH0013) is essential.

- **Prof Frank Smith** f.smith@ucl.ac.uk

The project(s) will be chosen from the following three areas:

1. *Industrial modelling problems such as in the internal and external flows of fluid associated with vehicle movements on land, sea or air*
2. *Biomedical flows such as through branching vessels or flexibly walled vessels*
3. *Modelling related to sports such as for balls, bouncing and vehicle movements*

   Pre-requisites: the projects above are suitable for students who have taken a full range of methods courses, have experience with the theory of fluids and are interested in ap plying mathematics.

- **Prof Valery Smyshlyaev** v.smyshlyaev@ucl.ac.uk

  1. *High frequency scattering: asymptotic methods and analysis*

     Problems of wave scattering are mathematically boundary value problems for a PDE. Their approximate solutions for high frequencies can be constructed analytically by a multivariable version of WKB method, which is one of asymptotic methods. Such approximations have a clear physical meaning, and tools of analysis are needed for controlling the accuracy of these approximations.

     Desirable but not essential pre-requisites: Waves and Wave Scattering (MATH0080) and Analysis 4 (MATH0051);

  2. *Multi-scale problems and homogenisation: asymptotic methods and analysis*

     Nearly everything around us contains multiple scales, i.e. has often invisible microscopically varying physical properties on which their visible macroscopic properties depend. Mathematically, one needs to deal with boundary-value problems for PDEs with microscopically varying coefficients, and then homogenisation becomes the process of deriving approximate PDEs with macroscopic coefficients. One way of doing this is via asymptotic methods with respect to the underlying small parameter, and the resulting approximations often display interesting physical effects. Tools of analysis are needed for controlling the accuracy of such approximations.

     Desirable but not essential pre-requisites: Functional Analysis (MATH0018) and Mathematical Methods 4 (MATH0056).

- **Dr Sergei Timoshin** s.timoshin@ucl.ac.uk

  1. *Two-fluid flows*

     Two-fluid flows can be studied in various approximations which reflect the specifics of the flow (e.g. thin layers), in two and three dimensions, with or without explicit time dependence. There are many interesting and unsolved problems related, for example, to flow separation and instability.

     Prerequisites: Knowledge of fluid dynamics at the level of Real Fluids (MATH0077) is essential.

- **Prof Jean-Marc Vanden-Broeck** j.vanden-broeck@ucl.ac.uk

  1. *Analytical and numerical studies of waves of large amplitude*

     The project is concerned with studies of waves propagating at the surface of a fluid. It is a free surface flow problem because it involves solving equations in a domain whose shape has to be found as part of the solution (the shape of the upper surface of the fluid is one of the unknows). Analytical methods (based on asymptotic expansions) and numerical methods will be reviewed. As time permits new problems will be considered.

     Pre-requisites: Fluid Mechanics (MATH0015) or equivalent.

- **Prof Helen Wilson** helen.wilson@ucl.ac.uk

1. *Mathematical modelling of Atomic Force Microscopy in the presence of liquid*

   In atomic force microscopy (AFM), a probe particle is attached to a force transducer (essentially a sprung lever) and brought close to the surface to be measured.

   The theory is quite simple: the force measured by the lever tells us how far upwards the particle has been displaced, which allows the scientist (by moving the probe parallel to the plane) to determine the shape of the surface.

   UCL engineers, however, are interested in using the AFM to measure something more. They want to measure an attractive force between the probe particle and the surface. And worse: they need to do this in the presence of a viscous fluid.

   There are now multiple forces applied to the probe particle. Can we disentangle them through mathematical modelling?

   This project should involve minimal fluid mechanics, but some interesting mathematical modelling and potentially the ability to solve dynamical systems, with genuine real-world applications.

2. *Jamming in concentrated suspensions*

   You've probably played with "oobleck" at some point: a concentrated mixture of cornflour and water that flows like a liquid if you stir it slowly, but shatters if you move the spoon too fast. This solid-like behaviour at fast speeds is an instance of the phenomenon of jamming. Modelling these jamming suspensions has been a hot topic of research for the last few years.

   In this project you will take a recent equation [1] which has been proposed to capture jamming phenomena, and try it out in a few different experimental situations. The simplest ones will be pen-and-paper exercises, but there will be some computation in the more complicated setups.

   [1] J J J Gillissen, C Ness, J D Peterson, H J Wilson, M E Cates. Constitutive model for time-dependent flows of shear-thickening suspensions. Physical Review Letters 123, 214504 (2019).

3. *Bead-spring constructions in viscous flow.*

   We often model viscoelastic fluids as a viscous fluid containing tiny bead-spring pairs (two beads connected by an elastic spring) [1]; and a very recent model involves three beads connected in a complex way by three springs [2]. But these are always idealised beads that don't interact fully with the fluid and each other.

   If two real spheres are suspended in a viscous fluid in an applied shear flow, they orbit around each other. These are called Jeffery orbits after the work that found something similar for ellipsoidal particles over a century ago [3].

   In this project you will use numerical simulations (code already available) to see how the presence of springs affects the orbits of pairs and triplets of particles under shear flow.

   [1] See, for instance, https://www.ucl.ac.uk/~ucahhwi/GM05/lecture4-5.pdf

   [2] J Eggers, T B Liverpool, & A Mietke. Rheology of Suspensions of Flat Elastic Particles. Physical Review Letters 131, 194002 (2023)

   [3] G B Jeffery. The motion of ellipsoidal particles immersed in a viscous fluid (1922) https://doi.org/10.1098/rspa.1922.0078 (1922)

- **Dr Edwina Yeo** edwina.yeo.14@ucl.ac.uk

1. *Mathematical modelling blood clot formation and treatment* Blood clot formation in arteries is the leading cause of heart attacks and strokes. The formation of blood clots is determined by

the speed of the blood flow, the number of platelets in the blood and the number of activation proteins. Blood clots can be treated by administering drugs which dissolve the clot - however this can also damage surrounding blood vessels. Mathematical models can be used to predict when and how to administer treatment.

This project will involve developing models for blood clot formation using continuous partial differential equations which track the clot size, the amount of platelets in the blood and the speed of the blood flow. The student will learn how to solve partial differential equations with moving boundaries (in this case the clot height), how to determine fluid flow in a blood vessel with an obstruction (using lubrication theory). This project has both analytical components and numerical components (no experience necessary), with the balance according to the student's interest.

Prerequisites: familiarity with fluid mechanics and viscous flows: MATH0015 Fluid Mechanics, MATH0077 Real Fluids, experience with partial differential equations and vector calculus e.g. MATH0016 Mathematical Methods 3. Desirable: experience in programming in any of the following: Python, Matlab, Julia is beneficial but not required.

3 Mathematical Physics

- **Prof Christian Boehmer** c.boehmer@ucl.ac.uk

  1. *Continuum mechanics with microrotations*

     The aim of this project is to study elasticity theory in the presence of micro-rotations. This theory is known under a few different names like Cosserat elasticity or Micropolar elasticity. As a first step the candidate would have to become familiar with elasticity theory (linear and non-linear) and next include micro-rotations. Various routes could be explored ranging from more computational work using Mathematica or more analytical work which would involve the calculus of variations to study equations of motion.

     Pre-requisites: No particular prerequisites.

  2. *Modified theories of gravity with diffeomorphism non-invariance*

     The first part of the project is to study the variational approach to the Einstein field equations and looking at the original Einstein action, sometimes called the Gamma squared action, which is different from the Einstein-Hilbert action commonly used. This can be used to set up a modified theory of gravity with second order field equations similar to those found in other popular modified gravity models. Interestingly, this model is no longer diffeomorphism invariant in general. The main part of the project is about studying this model in some concrete situations like cosmology, spherical symmetry or the study of gravitational waves. There are many avenues that can be explored further.

     Pre-requisites: Mathematics for General Relativity (MATH0025)

  3. *Interior solutions with spherical symmetry*

     The Schwarzschild interior solution is a well-known solution for the Einstein field equations where the source is an ideal fluid. Similar solutions can be constructed in other setups and this project will explore such interior solutions. This can be done for a variety of different models. Generally students will have to deal with systems of non-linear ODEs. Sometimes explicit solutions can be found. When this is not possible, one can use perturbation techniques, approximation methods or numerical solutions.

Pre-requisites: Mathematics for General Relativity (MATH0025)

4. *Azimuthal geodesics in cosmology (or An applied study of special functions)*
   Geodesics play an important role in cosmology, azimuthal geodesics are a special type of geodesics which naturally appear for certain cosmological models. This project studies this curves. No prior cosmology knowledge is required! The mathematics that unfolds when studying these curves involves various ODEs and integrals. To tackle these, one needs special functions and encounters elliptic integrals. The project is a mix of analytical and numerical work. At the end of the project students will have a good understanding of special functions and how they relate to certain questions in cosmology. Co-supervised with Betti Hartmann.
   Pre-requisites: Mathematics for General Relativity (MATH0025)

Pre-requisites: Please note that most of these projects require a good deal of programming in Mathematica. It is therefore essential that candidates have some programming background and are willing to invest effort into learning Mathematica.

- **Dr Selim Ghazouani** s.ghazouani@ucl.ac.uk

1. *Lorentzian manifolds*
   Lorentzian geometry is a generalisation of Riemannian geometry that is the conceptual framework for Einstein's relativity. While the theory is formally very similar to its Riemannian counterpart, it is a very different world altogether. For instance, not every manifold carries a Lorentzian structure. In this project, the student will study the interplay between the topology of manifolds and the Lorentzian geometry, starting with the following question: which two and three-dimensional manifolds carry a Lorentzian structure?
   Pre-(or Co-)requisite: MATH0072 Riemannian Geometry
2. *Generic dynamics*
   A dynamical system is the datum of a transformation of a space (be it a homeomorphism or a differential equation) which determines the evolution of a point as time goes by. They are the mathematical formalisation of many a physical phenomenon, such as the evolution of the solar system or a gas particle moving freely within a box.
   This project will centre around the following question: what does a typical dynamical system look like? Mathematicians have come up with many different examples of systems evolving in qualitatively drastically different ways, but somehow experience shows that only a handful of them can actually be observed in nature. In particular we will discuss formal conjectures of Smale from the 70s putting forward a conceptual explanation for this phenomenon, and potentially more recent developments in the field of generic dynamics.

- **Prof Rod Halburd** r.halburd@ucl.ac.uk

1. *Topics in complex analysis*
   Pre-requesite: MATH0013 Complex Analysis.
   Examples of projects include: value distribution of entire and meromorphic functions; Riemann surfaces and Riemann theta functions; differential and functional equations in the complex domain; Riemann-Hilbert problems; conformal and quasi-conformal mappings; approximation theory; analogues of complex analysis in other settings (discrete complex analysis, discrete holomorphic functions on graphs, analysis over the quaternions); applications of complex

analysis to mathematical physics (e.g., theta functions and finite-gap potentials in quantum mechanics, discrete holomorphic functions and the Ising model of ferromagnetism in statistical mechanics).

2. *Topics in general relativity*

   Pre-requesite: MATH0025 Mathematics for General Relativity.

   Examples of projects include: symmetry and conserved quantities in general relativity; approximation methods; matter sources for vacuum metrics; coordinate and spacetime singularities; gravitational waves; exact solutions; electromagnetic fields in vacuum.

3. *Rotating starts and galaxies*

   There are no specific pre-requisites for this project beyond core mathematics modules.

   We will look at the mathematical theory of rotating liquid masses subject only to their own gravitational fields. The non-relativistic theory was developed by Newton, Jacobi, Dirichlet, Dedekind, Riemann, Poincaré, Jeans, E. Cartan, Chandrasekhar and Lebovitz. We will examine the different kinds of configurations that are possible and the bifurcations that occur as the speed of rotation is increased. These are very important models in astrophysics. We can possibly consider the effects of adding electromagnetic fields or relativistic effects.

- **Dr Betti Hartmann** ucahbha@ucl.ac.uk

1. The student would be working on a project related to nonlinear effects in classical field theory. The resulting coupled differential equations need (usually) to be solved numerically. The student would be provided with tools to do so. Being able to program in FORTRAN, or willing to learn, would be useful. Possible projects of current interest could be:

   a) Holographic superconductors with competing order parameters

   b) Black holes and compact objects in extended gravity models

2. Black holes and other compact objects are accreting matter from their environment which leads to observable and quantifiable effects. The student would be investigating accretion processes around compact objects such as black holes and neutron stars by using large scale simulations and studying the observable effects. Good numerical skills would be very useful.

3. *Azimuthal geodesics in cosmology (or An applied study of special functions)*

   Geodesics play an important role in cosmology, azimuthal geodesics are a special type of geodesics which naturally appear for certain cosmological models. This project studies this curves. No prior cosmology knowledge is required! The mathematics that unfolds when studying these curves involves various ODEs and integrals. To tackle these, one needs special functions and encounters elliptic integrals. The project is a mix of analytical and numerical work. At the end of the project students will have a good understanding of special functions and how they relate to certain questions in cosmology. Co-supervised with Christian Boehmer.

   Pre-requisites: Mathematics for General Relativity (MATH0025)

4. *Azimuthal geodesics in cosmology (or An applied study of special functions)*

   Geodesics play an important role in cosmology, azimuthal geodesics are a special type of geodesics which naturally appear for certain cosmological models. This project studies this curves. No prior cosmology knowledge is required! The mathematics that unfolds when studying these curves involves various ODEs and integrals. To tackle these, one needs special functions and encounters elliptic integrals. The project is a mix of analytical and numerical work. At the end of the project students will have a good understanding of special functions and how they relate to certain questions in cosmology. Co-supervised with Christian Boehmer.

Pre-requisites: Mathematics for General Relativity (MATH0025)

- **Dr Michal Kwasigroch** m.kwasigroch@ucl.ac.uk

  1. *Quantum wavefunction overlap in magnetic materials with localised electrons*

     Magnetism is an inherently quantum phenomenon that is associated with electronic spin – an intrinsic property carried by each electron that is a measure of its interaction with the magnetic field. When electronic spins are aligned we say that the material is magnetised. The project will focus on materials, where the magnetism is generated by mobile electrons that carry electric current as well as localised ones trapped by static ions. The interplay between the two types of electrons is responsible for a range of interesting phenomena, often referred to as Kondo Physics. One such phenomenon is the anomalous increase of a material's resistance as the temperature is lowered.

     The precise location as well as momentum of an electron cannot be specified. This concept is known as the Heisenberg Uncertainty Principle. Electrons are instead described by a complex wavefunction, in a similar way that we mathematically describe the ripples on the surface of water. The wavefunction measures the probability of finding a particle at a given point as well as its likely speed. Wavefunctions of localised electrons can have many different shapes, e.g. s or p orbitals. Localised electrons can also tunnel from one ion to another. The effectiveness of this tunnelling depends on the overlap between the electronic wavefunctions centred on different ions and is responsible for the alignment of their spins as well as magnetism. One of the aims of the project will be to calculate this overlap as well as the resulting tunnelling rate and magnetism.

     Prior knowledge of quantum Physics is highly desirable but is not an essential requirement for this project.

     References

     [1] R. Shankar, Principles of Quantum Mechanics

     [2] Lev Landau, Quantum Mechanics: Non-Relativistic Theory

     [1] Chapters 16 and 17 of P. Coleman, Introduction to Many-Body Physics

4 Numerical Analysis and Financial Mathematics

- **Prof Timo Betcke** t.betcke@ucl.ac.uk

  1. *Representation of electromagnetic fields through fundamental solutions*

     In many practical algorithms we need to numerically represent electromagnetic solutions through simple basis functions. In this project we want to focus on the representation of solutions of Maxwell equations through the use of fundamental solutions. Interesting questions here are convergence properties and low-rank approximations to compress field representations. This project involves significant programming and good Python knowledge is expected.

- **Dr Alejandro Diaz** alex.diaz@ucl.ac.uk

  1. *Unconstrained optimisation through rare event simulation*

     Rare event simulation is a numerical technique for estimating the probability of events with very low probability of occurrence. These events could be, for example, the failure of a large system or the price of an asset reaching a prescribed threshold. The main tool for simulating these events is Monte Carlo simulation. By construction, the design of the algorithm involved has to be such that the space of events is sampled as efficiently as possible, otherwise the process can be extremely expensive. One efficient way of doing this is to model a rare event as a set contained in a sequence of nested subsets and generate samples according to increasing partial thresholds. In this project, we model the problem of unconstrained optimisation as one of rare event simulation. This means that we aim at sampling from a neighbourhood of the argument that maximises a function. This could be, for example, a fitness or loss function used in machine learning. The project requires familiarity with basic Monte Carlo simulation and proficiency in coding. Matlab, Python and R are suitable programming languages.

- **Prof Erik Burman** e.burman@ucl.ac.uk

  1. *Recovery of the coefficient in a second order elliptic method using the Kohn-Vogelius method*

     In this project we are interested in recovering the coefficient in a second order elliptic problem. This coefficient is typically a material property and can allow for the inverse identification of a material by measurements of for example the temperature distribution. We will use a finite element method in the context of the Kohn-Vogelius penalty method for the reconstruction. The project is mainly computational but there may be scope for some theoretical investigations. We suggest that computations can be done in the finite element package FreeFEM (https://freefem.org), but students who prefer to work in Fenics, Ngsolve or some other package may do so.

- **Dr Max Jensen** max.jensen@ucl.ac.uk

  1. *Extending the Brezis-Ekeland Principle for Deep Learning: Application to Nonlinear Partial Differential Equations*

     This project aims to extend the work arXiv:2209.14115 of Carini, Jensen, and Nürnberg on applying the Brezis-Ekeland principle in deep learning to solve gradient flows, which arose from Laura Carini's MSc dissertation. The original paper proposed a deep learning method for numerically solving partial differential equations (PDEs) that arise as gradient flows using the Brezis-Ekeland principle. This principle naturally defines an objective function to be minimised, making it ideally suited for a machine-learning approach using deep neural networks.

     The proposed extension will apply this deep learning approach to a broader class of nonlinear PDEs. The project will explore the potential of the Brezis-Ekeland principle in handling more complex systems and investigate the method's performance in higher dimensions. The goal is to develop a robust and efficient numerical solver for a wider range of nonlinear PDEs using deep learning, contributing to the ongoing efforts to integrate machine learning techniques into traditional numerical analysis. The project will also involve a comprehensive study of the theoretical aspects of the method, including error analysis and convergence properties.

The results of this project could have significant implications for various fields where PDEs play a crucial role, such as physics, engineering, and finance.

5 Number Theory

- **Dr Cecilia Busuioc.** cecilia.busuioc@ucl.ac.uk

  1. *Periods of Modular Forms*

     The spaces of modular forms have been of interest to number theorists because they exhibit natural rational structures. In MATH0104, we saw that the vector space of modular forms of a given weight for the full modular group is spanned by modular forms with rational Fourier coefficients and its finite-dimensionality led to interesting identities with a wide-range of applications. The theory of Eicher-Shimura provides us with another rational structure coming from the periods of modular forms. The purpose of this project is to first understand the main theory and possibly look at some applications (e.g. in relation to binary quadratic forms, zeta-functions associated to real quadratic fields) and then to study some recent surprising results of D. Zagier and collaborators who show that once one assembles the Hecke eigenforms and their suitable period polynomials into a generating function, the result is a product of well-known theta functions. One could then further explore some consequences of this identity, such as recovering the Fourier coefficients of the Hecke eigenforms in question from the given identity, which the authors were only able to show for levels 2,3, and 5.

     Recommended pre-requisites: MATH0035(Algebraic Number Theory), MATH0104 (Modular Forms)

  2. *Modular Curves, Regulators of Siegel Units and Applications*

     In Number Theory, it is a classical approach to associate to an object of arithmetic significance an L-function defined by an Euler product encoding local information which then one hopes to relate to global, geometric objects. Conjectures of Zagier and Boyd are such examples in the case of an elliptic curve defined over the rationals. Recent work of F. Brunault gives us explicit formulas of regulators of Siegel units (these are units in the function field of a modular curve, which is the corresponding algebraic curve obtained from the quotient of the complex upper half plane by the action of a congruence subgroup) as Mellin Transforms of certain Eisenstein Series of weight 1, which can be used to provide numerical examples of the conjectures mentioned above. The goal of this project is to study Brunault's paper and possibly compute further numerical examples of the conjectural formulas.

     Recommended pre-requisites: MATH0036 (Elliptic Curves), MATH0104 (Modular Forms)

  3. *Cyclotomic Fields and Iwasawa Theory*

     Recommended Pre-requisites: MATH0021(Commutative Algebra), MATH0022 (Galois Theory), MATH0035(Algebraic Number Theory)

  4. *Other topics in Algebraic Number Theory and Arithmetic Geometry*

- **Prof Vladimir Dokchitser** v.dokchitser@ucl.ac.uk

  1. *Tate's thesis*

In his PhD thesis, Tate gave a new proof of the analytic continuation and the functional equation of Hecke L-functions (a generalisation of the Riemann zeta-function and of Dirichlet L-functions). His approached relied on developing Fourier theory for p-adic numbers and has had a vast impact on number theory. The aim of the project is to present Tate's proof and to illustrate it with well-chosen explicit examples.

Prerequisites: A good understanding of p-adic numbers is essential (MATH0034). Some familiarity with Fourier series (MATH0016), algebraic number theory (MATH0035) and Dirichlet L-functions (MATH0083 or MATH0061) is desirable. Tate formulates everything in terms of local fields, but the project can concentrate just on p-adic numbers (a local field is to a number field as $\mathbb{Q}_p$ is to $\mathbb{Q}$).

- **Dr Luis Garcia Martinez** luis.martinez@ucl.ac.uk

I am happy to discuss a variety of projects in number theory or algebra with students that have taken or plan to take at least three of the following modules: Algebraic Number Theory (MATH0035), Elliptic Curves (MATH0036), Modular Forms (MATH0104), Prime Numbers and their Distribution (MATH0083), Further Topics in Algebraic Number Theory (MATH0061).

Some examples of projects are below.

1. *p-adic numbers and quadratic forms*
   The p-adic numbers are a fundamental tool of modern number theory. The aim of this project would be to gain an understanding of them and some of their many applications to arithmetic questions.
2. *Special elements of number fields and special values of L-functions*
   There is a fascinating, mostly conjectural, connection between the units of certain number fields and the values of L-functions. The goal of this project would be to explore what is known for cyclotomic and imaginary quadratic fields, from a theoretical and possibly also from a computational point of view.

- **Prof Richard Hill** r.m.hill@ucl.ac.uk

1. *Topics in Number Theory*
   Prerequisites: the exact prerequisites will depend on which topic chosen, but you should have taken at least three of the modules Number Theory (MATH0034), Algebraic Number Theory (MATH0035), Elliptic Curves (MATH0036), Prime Numbers and their Distribution (MATH0083) by the end of the third year.

- **Dr Nikoleta Kalaydzhieva** n.kalaydzhieva@ucl.ac.uk

The polynomial Pell equation

For a given non-zero positive integer D, which is not a square, we define Pell's equation to be $x^2 - Dy^2 = 1$, and is classically solved in positive integers x and y. Moreover, we know that solutions always exist and there are infinitely many of them. In this project we would try to better understand the polynomial Pell's equation, where for a given $D(t) \in \mathbb{C}[t]$, we try to find polynomials with complex coefficients $x(t)$, $y(t)$. Do we always have solutions as in the classical case, and if so how many? We can also change our coefficient space and ask how that would change our problem.

Prerequisite: MATH0034 (Number Theory)

- **Prof Yiannis Petridis** i.petridis@ucl.ac.uk

  1. *Lattice counting problems in Euclidean and hyperbolic spaces*
  2. *Ergodic theory and Number Theory*
  3. *The Erdős-Kac theorem on the number of distinct prime factors of the natural number n*
  4. *Selberg's theorem on the normal distribution of the Riemann-zeta function on its critical line*

  Pre-requisites: Projects normally require Prime Numbers and their Distribution (MATH0083), and elementary probability. Depending on the project, Geometry and Groups (MATH0052) or Multivariable Analysis (MATH0019) may be useful.

- **Dr Ian Petrow** i.petrow@ucl.ac.uk

  I am happy to discuss projects with any students who have taken or will be taking Prime Numbers and their distribution (MATH0083) or Modular Forms (MATH0104) and prefer to tailor projects to students' individual interests. Please contact me to discuss possible projects in number theory! Nonetheless here are a few ideas to get started:

  1. *100% of Galois groups over Q are S_n.*

     When we study Galois theory, we learn to compute the Galois group of a polynomial, or more generally a finite extension of fields. The Galois group of a degree n irreducible polynomial is always a subgroup of the symmetric group S_n. It is natural to ask 'Which subgroups of S_n occur as Galois groups?', and if you just start to write down examples by picking a polynomial 'at random', you will find that you very often get the whole of S_n as its Galois group. In this project, you will make that idea precise and show that, if one orders polynomials of fixed degree with integer coefficients by the maximum absolute value of their coefficients, then, as size of the coefficients gets large, the proportion of polynomials with Galois group the full S_n approaches 100%. The proof of this fact is a beautiful mix of algebraic number theory, group theory, and prime number theory. A reach on this project would be to try to read and understand a recent breakthrough paper of Bhargava (Fields Medal).

     Prerequisites: Required: Galois Theory (MATH0022) and Number Theory (MATH0022).

     Recommended: Algebraic Number Theory (MATH0035) and Prime Numbers and their distribution (MATH0083).

  2. *Representation of integers by quadratic forms and modular forms*

     Let $Q(x)$ be a positive-definite quadratic form with integer coefficients in at least 3 variables. A basic and old question for each $n > 0$ is how many integral representations of n by the quadratic form $Q(x)$ are there? For some highly structured specific choices of Q, we can give an exact formula for the number of x in $\mathbb{Z}^r$ such that $n = Q(x)$, but in general an exact formula isn't possible. Instead, we look for approximate formulas for the number of solutions as $n \to \infty$. For quadratic forms in 3 or 4 variables the proof of such a formula uses modular forms, which are certain special functions on hyperbolic spaces which have deep connections to modern number theory. The goal of this project is to learn the proof of the asymptotic formula for the representation number and use this as motivation to learn the theory of modular forms. Of particular interest will be certain examples called theta functions, and their role in the proof of the representation number theorem.

Prerequisites: MATH0051 Analysis 4: Real Analysis MATH0052 Geometry and Groups MATH0083 Prime Numbers and their Distribution

3. *Moments of L-functions* The Riemann Zeta Function (of Riemann Hypothesis fame) is the ur example of an L-function, a class of holomorphic functions on $\mathbb{C}$ that have deep connections to number theory. They control the distribution of prime numbers and play a key role in the wide-ranging web of conjectures at the forefront of research in number theory called the Langlands Program. In this project, we study L-functions from a statistical/probabilistic point of view. Namely, we consider various "families" of L-functions and estimate power-averages (or, moments) of the L-functions over the family. The goal of this paper is to understand a paper of Sounararajan that is able to upper bound moments of L-functions assuming the Generalized Riemann Hypothesis, and a paper of Rudnick-Soundararajan that is able to lower-bound moments of L-functions.

- **Dr Alex Walker** alexander.walker@ucl.ac.uk

1. *The Congruent Number Problem*

   A congruent number is a positive integer which appears as the area of some right triangle with rational side lengths. For example, the (3,4,5) triangle has area 6 and demonstrates that 6 is a congruent number. The problem of classifying congruent numbers was partially resolved in 1983 by Tunnell's theorem, which related congruence to the behavior of a certain Diophantine equation related to half-integral weight modular forms. This project begins with the classic history of the congruent number problem and then discusses modern connections to elliptic curves and modular forms.

   Pre-requisites: Modular Forms (MATH0104) is required. Prime Numbers and their Distribution (MATH0083) and Elliptic Curves (MATH0036) are both recommended.

- **Prof Andrei Yafaev** a.yafaev@ucl.ac.uk

1. *Complex multiplication of elliptic curves* It is a curious fact that the (transcendental) number $e^{\pi\sqrt{163}}$ (known as the Ramanujan constant) is actually very close to an integer (its decimal expansion has twelve nines after the decimal point). This fact can be explained by a rather deep theory - that of complex multiplication of elliptic curves. An elliptic curve (over the complex numbers) has complex multiplication if its endomorphism ring is larger than just the integers. The first main theorem of complex multiplication of elliptic curves is that the j-invariant of such an elliptic curve is an algebraic integer. The aim of the project is to understand a proof of this theorem which relies on the study of the so-called 'modular polynomial'.

   Prerequisites: the Algebraic Number theory module and the Elliptic curves module.

6 Geometry and topology

- **Dr Dario Beraldo** d.beraldo@ucl.ac.uk

  Examples of projects (the actual project will be decided together with interested students):

  1. *Introduction to algebraic curves* We study algebraic curves and some fundamental formulas (Riemann-Hurwitz, Riemann-Roch, Hurwitz's automorphisms theorem). Alternatively, we could study the moduli space of curves.
  2. *Tsen's theorem, rationally connected varieties, Graber-Harris-Starr's theorem* We could study the several beautiful properties of rationally connected varieties and Mori's bend-and-break method.
  3. *Oriented cobordism theory and elliptic genera* Here we study Thom's theorem which identifies the oriented cobordism ring with a polynomial ring on the classes of even projective spaces. We will use it to describe elliptic genera and the modular form that goes with them.
  4. A project in (geometric) representation theory: e.g., an introduction to modular representation theory, or the Borel-Weil theorem.
  5. *Milnor fibration theorem and generalizations* For instance, the Deligne-Milnor formula in mixed characteristic, or the study of monodromy, b-functions, Igusa's zeta functions.

  Prerequisites for all: a solid foundation in algebra and/or geometry. For example: algebraic geometry, algebraic topology, representation theory, smooth manifolds, topology and groups.

- **Dr Aleksander Doan** a.doan@ucl.ac.uk

  General comments

  I will be happy to supervise a variety of projects on differential geometry and topology, as well as complex analysis, partial differential equations, and relationships between geometry and physics, depending on the student's background and interests. Below are some examples.

  1. *Differential forms and cohomology*

     The goal of this project is to learn about smooth manifolds, higher-dimensional analogues of curved shapes such as curves and surfaces, and the calculus of differential forms, which is a generalization of classical vector calculus. Differential forms appear in many contexts in geometry, analysis, and physics, and a classical theorem of de Rham relates them to cohomology groups, important topological invariants of manifolds.

     Prerequisites: MATH0051 Analysis 4, MATH0019 Multivariable Analysis

     Related modules: MATH0020 Differential Geometry, MATH0023 Algebraic Topology, MATH0072 Riemannian Geometry
  2. *Invariants of knots*

     A knot is an embedding of a circle inside the three-dimensional space and knot theory studies when one such embedding can be continuosly deformed to another. Basic tools for answering such questions are invariants of knots, that is numbers or algebraic objects which do not change under such continuous deformations. This project explores various classical invariants of knots, such as the Alexander polynomial, Seifert form, and knot signature, and their applications.

Prerequisites: MATH0023 Algebraic Topology

Related modules: MATH0074 Topology and Groups

3. *Vector bundles and characteristic classes*

   A vector bundle is a collection of vector spaces parametrized by points of a manifold, which can twist in a topologically nontrivial way as we travel inside the manifold, like the Mobius band which twists when we travel around it. Another example is the tangent bundle of a smooth manifold. A basic question in topology is to classify vector bundles on a given manifold. This project is about classifying spaces and characteristic classes which are powerful tools of algebraic topology that help us solve this problem. They turn out to be related in a fascinating way to differential forms known from analysis. The relationship between vector bundles, characteristic classes, and differential form is the foundation of the geometric interpretation of electromagnetism and other gauge theories in physics.

   Prerequisites: MATH0051 Analysis 4, MATH0019 Multivariable Analysis, MATH0023 Algebraic Topology

   Related modules: MATH0020 Differential Geometry, MATH0072 Riemannian Geometry

4. *Elliptic operators on manifolds*

   Elliptic operators are differential operators on manifolds generalizing the well-known operators known from vector calculus and complex analysis such as the Laplacian and the Cauchy-Riemann operator. In this project we will study general analytic properties of elliptic operator on manifolds, such as existence of parametric and elliptic regularity, which will force us to abandon the realm of smooth functions and venture into the world of distributions and Sobolev spaces. The final goal is to understand the Hodge decomposition theorem, which is one of the foundational theorems of modern geometry, relating analysis of partial differential equations to topology.

   Prerequisites: MATH0051 Analysis 4, MATH0019 Multivariable Analysis, MATH0018 Functional Analysis

   Related modules: MATH0020 Differential Geometry, MATH0072 Riemannian Geometry, MATH0070 Linear Partial Differential Equations, MATH0090 Elliptic Partial Differential Equations

5. *Riemann surfaces and the uniformization theorem*

   Riemann surfaces lie at the crossroads of algebraic geometry, differential geometry, and complex analysis. The goal of this project is to understand Poincare's uniformization theorem. This foundational results in the study of Riemann surfaces asserts that all only simply-connected Riemann surfaces are equivalent to the disk, the complex plane, or the sphere. An alternative statement of this theorem is that every Riemann surface admits a Riemannian metric of constant curvature. The project will explore both the complex and Riemannian sides of the theorem and the relationship between them.

   Prerequisites: MATH0013 Analysis 3, MATH0051 Analysis 4, MATH0020 Differential Geometry

   Related modules: MATH0072 Riemannian Geometry, MATH0070 Linear Partial Differential Equations, MATH0090 Elliptic Partial Differential Equations, MATH0036 Elliptic Curves, MATH0074 Topology and Groups, MATH0052 Geometry and Groups

- **Prof Francis Johnson** f.johnson@ucl.ac.uk

I offer a wide variety of projects in and around the areas of Algebraic Topology, Homological Algebra, Group Representation Theory and Discrete Subgroups of Lie groups. Typically these might include :

1. *Fibre bundles and spectral sequences*
2. *Lefschetz complexes and Poincaré duality*
3. *Borel density and Mostow rigidity*

- **Dr Mikhail Karpukhin** m.karpukhin@ucl.ac.uk

  I am happy to discuss any projects on functional analysis, differential geometry or spectral theory. Some examples are below:

  1. *Minimal surfaces in the sphere and Euclidean space*

     Minimal surfaces are mathematical models of soap films – they are surfaces that locally minimise the area. Even in the simplest case of surfaces inside $\mathbb{R}^n$ or $\mathbb{S}^n$ there are many questions that remain unsolved. The project is devoted to exploration of various constructions of minimal surfaces and their connection to the study of eigenvalues of the Laplace operator.

     Prerequisites: Differential geometry (MATH0020)

- **Prof Ed Segal** e.segal@ucl.ac.uk

  Topics in Geometry, Topology and Algebra:

  I'm happy to discuss potential projects in algebraic geometry, differential geometry, algebraic topology or algebra. A couple of examples are below.

  1. *Principal bundles*

     Pre-requisites/related courses: Differential Geometry (MATH0020), Topology and Groups (MATH0074), Riemannian Geometry (MATH0072).

  2. *Higher Ext groups*

     Pre-requisites/related courses: Homological algebra (MATH0021), Commutative Rings and Algebras (MATH0108), Representation Theory (MATH0073).

- **Prof Michael Singer** michael.singer@ucl.ac.uk

  1. *Geometry of Classical and quantum mechanics*

     In this project we shall explore geometric quantization: symplectic geometry is the correct setting for classical mechanics. Geometric quantization is a recipe (though more of an art-form) for constructing the Hilbert spaces of quantum theory starting from a symplectic manifold.

     Pre-requisite: Multivariable calculus (MATH0019), desirable: Differential geometry (MATH0020). Useful: Analytical Dynamics (MATH0054).

  2. *Other projects in differential geometry*

7 Analysis

- **Dr Shane Cooper** s.cooper@ucl.ac.uk

  1. *Mathematical approach to innovative composite material design*

     In this project we shall study solutions of second-order partial differential equations with rapidly oscillating coefficients using asymptotic analysis. The equations of interest arise from mathematical models for the behaviour of modern advanced man-made composite materials.

     Recommended pre-requisites are (not necessarily all of) the following: MATH0027 (Methods 5), MATH0078 (Asymptotic methods and Boundary layer theory), MATH0080 (waves and wave scattering), MATH0070 (linear PDE), MATH0018 (Functional Analysis) and MATH0071 (Spectral theory).

- **Dr Mahir Hadzic** m.hadzic@ucl.ac.uk

  1. *Phase Mixing and Landau damping*

     The aim of the project is to rigorously describe the phase mixing mechanism which is at the heart of the celebrated Landau damping phenomenon. Landau damping refers to the tendency of plasmas, as described by the Vlasov-Poisson system to equilibrate asymptotically in time. This is a mathematically interesting feature of the problem, as he equation has no manifest dissipation built in. The responsible mechanism is phase mixing. The project requires a good background in analysis.

  2. *Wave equations outside obstacles*

     We consider the wave equation outside a compact obstacle. The goal is to understand the decay-in-time properties of the solution assuming suitable boundary conditions on the boundary of the obstacle. We shall consider the Dirichlet, the Neumann, and the Robin boundary conditions. Our starting point is the seminal work of Morawetz from 1960's which relies on the so-called multiplier / vector-field method.

     Prerequisites: Analysis 4 (MATH0051), Recommended: Multivariable Analysis (MATH0019).

- **Prof Dave Hewett** d.hewett@ucl.ac.uk

  1. *Fractals, measure and integration*

     Description: In this project we will study the theory of measure and integration on fractals. Fractals are fascinating geometrical shapes possessing detail on every length scale - well-known examples include the von Koch curve and the Sierpinski gasket. Such sets typically have non-integer "dimension", and the notions of "measure" (length, area etc) that we are used to when dealing with integer-dimensional sets like curves and surfaces have to be generalised to deal with fractal sets. For instance, the Koch curve has infinite length (1-dimensional measure) but zero area (2-dimensional measure), since its fractal dimension $\log(4)/\log(3)$ lies strictly between 1 and 2. The student will learn about the classical Hausdorff measures and Hausdorff dimension, as well as more exotic self-similar measures defined on fractal attractors of iterated function systems. The aim will be to investigate properties of integration with respect to such measures, reviewing recently derived representation formulas for singular integrals on fractals, and attempting to generalise these to new scenarios not previously explored.

Prerequisites: MATH0051 Analysis 4 essential, MATH0017 Measure Theory strongly recommended

- **Dr Ilia Kamotski** i.kamotski@ucl.ac.uk

  1. *Topics in homogenisation theory*

     Prerequisite: Linear Partial Differential Equations (MATH0070)

- **Dr Mikhail Karpukhin** m.karpukhin@ucl.ac.uk

  I am happy to discuss any projects on functional analysis, differential geometry or spectral theory. Some examples are below:

  1. *Eigenvalues of the Laplace operator*

     The Laplace operator is a fundamental operator acting on an infinite-dimensional space of functions, but in many situation it has eigenvalues and eigenvectors just like any self-adjoint operator in linear algebra. We can look at various properties of Laplace eigenvalues, e.g. isoperimetric inequalities (which answer the question: which shape optimises a certain eigenvalue among all shapes of the fixed volume?), heat kernel (given an initial distribution of temperature in a room, how does it change over time?) or Weyl's law (how do eigenvalues distribute on the real line as they become large? Is there an approximate formula for the k^{th} eigenvalue?)

     Prerequisites: at least one of Functional Analysis MATH0018 or Measure Theory MATH0017

- **Dr Beatriz Navarro Lameda** beatriz.navarro@ucl.ac.uk

  1. *Discrete Dynamical Systems: Different Definitions of Chaos*

     There are many different definitions of chaos; i.e., of what it means for a function $f : X \to X$ from a compact metric space to itself to be chaotic. These definitions are not equivalent in general but they all capture the same basic idea of unpredictability or instability: it is not enough to know the trajectory of one point in order to predict the trajectories of other nearby points. In this project we will study several commonly encountered definitions of chaotic systems and their properties, and how these different notions are related to one another.

     Pre-requites: MATH0051 Analysis 4: Real Analysis

- **Prof Nadia Sidorova**. n.sidorova@ucl.ac.uk

  1. *Topics in Probability*

     Prerequisite: MATH0069 Probability

- **Dr Iain Smears** i.smears@ucl.ac.uk

  1. *Analysis of Partial Differential Equations arising in stochastic optimal control and game theory*

     There are many close connections between partial differential equations (PDE) and stochastic processes. In particular, models of stochastic optimal control and stochastic dynamic games can often be formulated in terms of PDE. Famous examples include the Fokker-Planck equation for the evolution of the probability density of a stochastic process, and the Hamilton-Jacobi-Bellman equation for the value function of a stochastic optimal control problem. There are also applications in Game Theory, such as Mean Field Games for games involving players subject to stochastic dynamics. This project will involve the mathematical analysis of the resulting PDE from one or more application areas. Using tools from functional analysis, measure theory, and function spaces, we can analyse the existence and uniqueness properties of various problems. The project may go in various directions; for instance, the analysis of Fokker-Planck equations, or Hamilton-Jacobi-Bellman equations, or the analysis of systems of mean field games from stochastic processes. Other possibilities include a study of the Kakutani-Fan-Glicksberg theorem and its application to the analysis of partial differential inclusions.

     Prerequisites for this project are: MATH0017 Measure theory, MATH0092 Variational Methods for Partial Differential Equations, MATH0018 Functional Analysis.

- **Prof Valery Smyshlyaev** v.smyshlyaev@ucl.ac.uk

  1. *High frequency scattering: asymptotic methods and analysis*

     Problems of wave scattering are mathematically boundary value problems for a PDE. Their approximate solutions for high frequencies can be constructed analytically by a multivariable version of WKB method, which is one of asymptotic methods. Such approximations have a clear physical meaning, and tools of analysis are needed for controlling the accuracy of these approximations.

     Desirable but not essential pre-requisites: Waves and Wave Scattering (MATH0080) and Analysis 4 (MATH0051);

  2. *Multi-scale problems and homogenisation: asymptotic methods and analysis*

     Nearly everything around us contains multiple scales, i.e. has often invisible microscopically varying physical properties on which their visible macroscopic properties depend. Mathematically, one needs to deal with boundary-value problems for PDEs with microscopically varying coefficients, and then homogenisation becomes the process of deriving approximate PDEs with macroscopic coefficients. One way of doing this is via asymptotic methods with respect to the underlying small parameter, and the resulting approximations often display interesting physical effects. Tools of analysis are needed for controlling the accuracy of such approximations.

     Desirable but not essential pre-requisites: Functional Analysis (MATH0018) and Mathematical Methods 4 (MATH0056).

- **Prof Alex Sobolev** a.sobolev@ucl.ac.uk

  1. *Pseudo-differential operators*

Pseudo-differential operators (PDO's) are generalisations of the familiar differential operators. Theory of PDO's forms a tremendously important part of modern Analysis. PDO's are used in Differential Equations, Mathematical Physics, Differential Geometry and many other domains. The aim of the project is to understand the basics of the PDO theory starting with the Fourier transform, PDO calculus and ending with the conditions that guarantee the boundedness of PDO's as linear operators.

Prerequisites: Analysis 4 (MATH0051), Functional Analysis (MATH0018), Measure Theory (MATH0017) is helpful but not critical.

2. *Mathematical theory of wavelets*

   The goal of the theory is to find a function on the real line such that the set of its translates and its rescaled copies forms a basis of $L^2(\mathbb{R})$, the space of square integrable functions. Such a function is called a wavelet. The aim of the project is to work through the Multiresolution Analysis which constitutes the basis of the whole approach, and to understand some known examples of wavelets.

   Prerequisites: Analysis 4 (MATH0051), Functional Analysis (MATH0018).

- **Prof Dmitri Vassiliev** d.vassiliev@ucl.ac.uk

1. *Spectral problems on Riemannian 3-manifolds*

   Consider a connected oriented closed Riemannian 3-manifold. There are three main differential operators acting on such a manifold: the Laplace-Beltrami operator, the operator curl and the (massless) Dirac operator. The project concerns the study of eigenvalues of these operators.

   There are two obvious cases when eigenvalues can be evaluated explicitly: torus equipped with Euclidean metric and round sphere (sphere equipped with standard metric, i.e. metric obtained by restriction of 4-dimensional Euclidean metric). But there are also special non-trivial metrics for which eigenvalues can be evaluated explicitly. The aim of the project is to examine some of these special cases.

   Pre-requisites: Functional Analysis (MATH0018) and Multivariable Analysis (MATH0019). Concurrent enrolment in Spectral Theory (MATH0071) would also be desirable.

8 Combinatorics

- **Dr Samuel Coskey** s.coskey@ucl.ac.uk

I am happy to discuss projects in Set Theory, Logic, Graph Theory, and Combinatorics. Here are some suggestions related to my own recent interests.

1. *Learn topics in Borel equivalence theory.* This area of set theory seeks to compare the complexity of equivalence relations. The equivalence relations can represent classification problems in mathematics, such as the isomorphism equivalence relation on a class of objects. The field involves methods from analysis, topology, set theory, and more.

   Reference: Gao, Su. Invariant Descriptive Set Theory. Chapman and Hall/CRC, 2008

Reference: Kanovei, Vladimir. Borel Equivalence Relations: Structure and Classification. AMS University Lecture Series, 2008

Useful experience: Analysis, Logic

2. *Study conjugacy in automorphism groups.* When studying the automorphism group of a structure, it is natural to classify the automorphisms up to conjugacy. One may study the conjugacy equivalence relation on the automorphism group of countable graphs, groups, and other structures. Apply tools from Borel equivalence theory to draw conclusions.

   Reference: The conjugacy problem for automorphism groups of countable homogeneous structures. https://arxiv.org/abs/1406.6411

   Useful experience: Algebra, Graph Theory

3. *Generalise the theory of group dynamics to higher cardinality.* There is a significant theory of Polish groups and their actions. The Polish topology means the groups are the same size as $\mathbf{R}$ and share the same Borel structure. An example of such a group is S_∞, the symmetric group on $\mathbf{N}$. What about for larger groups, for instance S_κ, the symmetric group on an uncountable cardinal κ? Much of the classical theory should generalize to larger cardinalities, but there will be some important differences.

   Reference: Gao, Su. Invariant Descriptive Set Theory. Chapman and Hall/CRC, 2008

   Reference: Generalized Baire spaces. https://arxiv.org/pdf/1310.6685.pdf

   Useful experience: Set Theory, Algebra, Point Set Topology

4. *Suggest another topic in Logic, Set Theory, Graph Theory, or Combinatorics.* If you have some ideas for a project that would interest you, please bring it forward and I can support you!

- **Dr Freddie Illingworth** f.illingworth@ucl.ac.uk

1. *Colouring triangle-free graphs*

   Every graph of maximum degree D can be coloured (greedily) with at most $D + 1$ colours. If the graph does not contain a triangle, then we can do much better, using only $D/\log(D)$ colours. This was a beautiful breakthrough using the idea of random colourings and is strongly related to the Ramsey number $R(3, t)$.

   A project in this area would start with the simpler case of independent sets in triangle-free graphs and could then go in a number of directions e.g. to colourings, Ramsey numbers, variants of triangle-free graphs, the Hard Core model.

   Prerequisites: MATH0029 (Graph theory and Combinatorics). Useful: MATH0057 (Probability and Statistics) or another module in probability.

- **Dr Mikhail Karpukhin** m.karpukhin@ucl.ac.uk

I am happy to discuss any projects on functional analysis, differential geometry or spectral theory. Some examples are below:

1. *Geodesic nets and eigenvalues of graphs*

   In this project we explore relation between two seemingly unconnected fields. First is spectral graph theory, widely used in computer science to efficiently encode various properties of large graphs. Second is geodesic nets, a geometric notion describing the shortest way to connect together a collection of points on a surface. It turns out that graphs optimising certain functionals from spectral graph theory have a canonical realisation as geodesic nets on spheres. In this project we will investigate this connection starting from the simplest graphs.

Prerequisites: Algebra 3 (MATH0014), Differential geometry (MATH0020).

- **Dr Lars Louder** l.louder@ucl.ac.uk

  1. *Stallings' theorem on groups with infinitely many ends and groups of cohomological dimension 1*
  2. *Diophantine problems in the free group, or 10,000 ways to write a commutator*

     The word $w = uuvuvvUUVUVV$ in the free group $< u, v >$ can be written as a commutator in two essentially distinct ways: $w = [uuvuv, vUU] = [uuvu, vvU]$ (check it!). It turns out that any commutator can be written, up to some natural equivalence which comes from the topology of compact surfaces, in only finitely many ways, and up to a slightly coarser, but still natural, equivalence, in at most 10,000 ways.

     The aim of the project will be to use this as an introduction to studying equations over the free group. A more geometrically inclined student could push this project in the direction of algebraic geometry over groups, and a student who leans towards computer science and is already a capable coder could feasibly attempt to reduce the bound of 10,000, possibly all the way down to 2 (I hope!), using linear programming and branch and bound techniques a la the proof of the Kepler conjecture.

  Pre-requisites: Topology and Groups (MATH0074). Recommended: Geometry and Groups (MATH0052).

- **Dr Alexey Pokrovskiy** a.pokrovskiy@ucl.ac.uk

  1. *Games on graphs* A positional game is one where two player take turns picking positions on a board with some rule to determine the winner. A simple example is tic-tac-toe where the board is a 3 by 3 grid, and the first player to claim three positions in a row/column/diagonal is the winner. Positional games can also be played on graphs — here the board is the set of all possible edges in a complete graph and the players take turns claiming edges. Various types of games like this have been studied with different win conditions. A common game is the "maker-breaker" game. Here the first player wants to create a copy of some fixed subgraph, while the second player wants to prevent them from doing this. This project would be about a variant of this called "saturation games", for example about some of the problems from the following paper:

     [1] Spiro, Sam. "Saturation games for odd cycles." arXiv:1808.03696 (2018).
  2. *Other problems about extremal combinatorics and graph theory* Other projects similar to the above are possible, for example on the topics of "twin-width of graphs" or "sublinear expanders".

     These projects are all suitable to students who have taken Graph Theory and Combinatorics MATH0029 or Combinatorial Optimization MATH0028.

- **Prof Alan Sokal** a.sokal@ucl.ac.uk

  Continued fractions in enumerative combinatorics

  Continued fractions arise in number theory as representations of irrational numbers. Here we are concerned, by contrast, with continued-fraction representations of (possibly divergent) power series. For instance, Euler proved in 1746 that

$$\sum_{n=0}^{\infty} n!\, t^n \;=\; \cfrac{1}{1 - \cfrac{1t}{1 - \cfrac{1t}{1 - \cfrac{2t}{1 - \cfrac{2t}{1 - \cfrac{3t}{1 - \cfrac{3t}{1 - \cdots}}}}}}} \, .$$

  This identity makes sense as a *formal power series*, even though the left-hand side is divergent for all $t \neq 0$. Since $n!$ counts the permutations of an n-element set, it is natural to want to refine this continued fraction by counting permutations with respect to some combinatorially interesting statistics (for instance, the number of cycles). Many such formulae have been found by enumerative combinatorialists in the last few decades. This project would involve learning about this work, and possibly finding new examples by computational exploration.

  Prerequisites: Complex Analysis (MATH0013) and the elementary theory of polynomial and formal-power-series rings (MATH0014) all come in. Some knowledge of combinatorics (e.g. MATH0029) would also be useful. A good background reference in enumerative combinatorics is Wilf's *Generatingfunctionology*. Finally, a good knowledge of MATHEMATICA will be essential to the computational side of this project.

- **Dr John Talbot** j.talbot@ucl.ac.uk

  1. *Counting cliques in graphs*

     How many triangles must a graph of given order and size contain? Recently [3] a very general version of this question was answered. This highly technical result was the culmination of many decades of progress on this topic starting with Rademacher in 1941. A project in this area could take many different directions, including looking at special cases and related algorithmic questions.

     [1] Nikiforov, Vladimir. The number of cliques in graphs of given order and size. Trans Am Math Soc. (2007) 363. 1599-1618

     [2] Razborov, Alexander. On the Minimal Density of Triangles in Graphs. Combinatorics, Probability and Computing, 17(4), (2008), 603-618

     [3] Reiher, Christian. The Clique Density Theorem. Annals of Mathematics, vol. 184, no. 3, (2016), pp. 683–707. Second Series.

     Prerequisites: MATH0029 (Graph Theory and Combinatorics).

  2. *Formalising extremal and probabilistic combinatorics in the Lean Proof Assistant*

     This project would involve choosing a recent result in combinatorics, understanding its proof and then formalising the result in Lean.

For an introduction to Lean see the Natural Numbers Game.

Please contact me directly to discuss possible choices of result to formalise.

9 Algebra

- **Prof Francis Johnson** f.johnson@ucl.ac.uk

  I offer a wide variety of projects in and around the areas of Algebraic Topology, Homological Algebra, Group Representation Theory and Discrete Subgroups of Lie groups. Typically these might include :

  1. *Projective modules and Algebraic K-Theory*
  2. *Projective resolutions and the syzygetic approach to module cohomology*
  3. *Representation theory of finite groups over* $\mathbb{Q}$ *and* $\mathbb{Z}$

- **Dr Ruth Reynolds** ruth.reynolds@ucl.ac.uk

  This year I will be offering projects in abstract algebra, in particular projects in noncommutative ring theory. Many of the concepts from commutative algebra become more complicated, and therefore more interesting, in this noncommutative world, which forms an active area of current research with applications to other areas of maths. If you are curious about this area in general, I highly recommend the following introduction written by Professor Chelsea Walton, which gives an excellent primer to the subject: `https://arxiv.org/abs/1808.03172` with accompanying video: `https://www.youtube.com/watch?v=G2ZX0Zq0BxM`.

  If you have taken modules such as Algebra 4 or Commutative Algebra, and are considering taking Representation Theory, then you will find any of these projects interesting. Of course, the projects can be tailored to suit your individual interests.

  1. *Skew Extensions and Noetherianity.* A skew extension is a quick way to make a noncommutative ring by taking a commutative ring and building a noncommutative structure based on this in a reasonably controlled way. Noetherianity, named after the famous mathematician Emmy Noether, is a fascinating 'finiteness' property which many algebraic structures can possess, but it can be very difficult to determine when an algebraic structure is Noetherian. In this project, we will explore skew extensions of various forms and their associated noetherianity properties.
  2. *AS regular Algebras.* One of the first interesting examples of a commutative ring is a polynomial ring. In this project, we will explore AS regular algebras which are thought of as 'noncommutative' polynomial rings. Our first aim will be to understand why they are viewed this way, and to explore 2 dimensional AS regular algebras which turn out to have a very elegant classification. There is a great deal of cutting-edge research involving AS regular algebras, and there are a number of links to other areas of maths such as algebraic geometry which may be explored.
  3. *Goldie's Theorem.* In commutative algebra, you have met the notion of a field of fractions of a ring such as how $\mathbb{Q} = \mathrm{Frac}(\mathbb{Z})$. One question you may be curious to answer, is how one might extend this notion to a noncommutative setting. In this project, we will explore

what it means to be a noncommutative fraction and we will discover that this is a very subtle notion. For example, if your ring is a commutative domain then it is a fact that it has a field of fractions, however if we generalise this to noncommutative rings then this fact no longer holds. The first big step towards figuring out when noncommutative rings can have a field of fractions was achieved by Goldie, and one aim of this project is to understand Goldie's theorem and its implications.

Suggested prerequisites: Algebra 4, Commutative algebra.

Suggested References:

Goodearl and Warfield, An introduction to noncommutative Noetherian rings.

D. Rogalski, An Introduction to Noncommutative Projective Geometry.

- **Dr Mark Roberts** m.l.roberts@ucl.ac.uk

  Projects in non-commutative ring theory. Possible topics include:

  1. *Unique factorisation domains*

     You have seen examples of commutative unique factorisation domains (UFDs), starting in Algebra 2 with Z and a few examples of algebraic rings of integers such as the Gaussian integers $Z[i]$, continuing in Algebra 3 with the polynomial ring $k[x]$. There are more examples in MATH0021 Commutative Algebra and MATH0035 Algebraic Number Theory. There are various ways of generalising the idea of unique factorisation to non-commutative rings and this project looks at the method described in Cohn, *Free Rings and Their Relations.* (This is a very tough read, so don't get put off if you have a look at it!) There is a quite interesting connection with embedding semigroups in groups and rings in fields (see second project) and one could look at the question: if R is a (non-commutative) UFD, does R^*, the semigroup of non-zero elements of R under multiplication, embed in a group?

  2. *Skew fields of fractions*

     This also generalises an idea familiar from commutative algebra: one can embed Z in its field of fractions Q and $k[t]$ in its field of fractions $k(t)$. More generally any commutative integral domain R there is a unique (up to isomorphism) field Q such that R embeds in Q and Q is generated as a field by the image of R (constructed in almost exactly the way as one creates Q from Z). Perhaps surprisingly, the corresponding results fails for non-commutative integral domains: there exists non-commutative integral domains which do not embed in any (skew) field and also non-commutative IDs which embed in non-isomorphic skew fields. The first example of an ID without a skew field of fractions was given by Mal'cev. This project looks at a way of analysing the embedding of rings in skew fields given in Cohn, *Free Rings and Their Relations* (same comment as above), using ideas of matrix ideals.

The pre-requisites for both projects are Algebra 4 (MATH0053) and at least two further algebra modules e.g. Commutative Algebra (MATH0021), Galois Theory (MATH0022), Algebraic Topology (MATH0023), Algebraic Number Theory, (MATH0035), Representation Theory (MATH0073). Commutative Algebra is particularly relevant.

Both projects involve reasonably abstract algebra but also involve quite a bit of algebraic calculations.

- **Prof Ed Segal** e.segal@ucl.ac.uk

  Topics in Geometry, Topology and Algebra:

  I'm happy to discuss potential projects in algebraic geometry, differential geometry, algebraic topology or algebra. A couple of examples are below.

  1. *Principal bundles*

     Pre-requisites/related courses: Differential Geometry (MATH0020), Topology and Groups (MATH0074), Riemannian Geometry (MATH0072).

  2. *Higher Ext groups*

     Pre-requisites/related courses: Homological algebra (MATH0021), Commutative Rings and Algebras (MATH0108), Representation Theory (MATH0073).

- **Dr Isidoros Strouthos** i.strouthos@ucl.ac.uk

  Projects involving topics related to abstract algebra and/or topology

  Such projects may involve material covered in modules such as 'Algebra 3: Further Linear Algebra' and 'Algebra 4: Groups and Rings' (as well as, perhaps, some material involved in one or more of the modules in 'Algebraic Topology', 'Homological Algebra', 'Commutative Rings and Algebras', 'Representation Theory'); you are more than welcome to contact me directly for more information regarding possible relevant projects.

- **Dr Matthew Towers** m.towers@ucl.ac.uk

  1. *Universal enveloping algebras*

     There is a ring U, called the 'universal enveloping algebra', associated to each Lie algebra L. The project will study the universal enveloping algebras associated to the special linear Lie algebra $\mathfrak{sl}(n)$ for $n = 2$ or 3 over fields of characteristic 2. The specifics will depend on the interests and the background of the student undertaking the project: for example, we might look at the centres of these algebras, their algebras of derivations, or their cohomology.

     Pre-requisites: Algebra 4 (MATH0053), and some of Commutative Algebra (MATH0021), Algebraic Topology (MATH0023), Representation Theory (MATH0073). Recommended concurrently: Lie groups and Lie algebras (MATH0075).

  2. *Category theory and topology in functional programming*

     One possible project begins by giving an introduction to category theory in the context of functional programming. After that it would investigate semantic approximation order, recursive definitions as fixed points, or monads and their algebras.

     Another idea would be to give an account of some remarkable work of Martin Escardo `https://www.cs.bham.ac.uk/~mhe/.talks/popl2012/escardo-popl2012.pdf` on decidable equality for function types.

     Pre-requisites: some programming skills, and knowledge of or willingness to learn a functional language, e.g. Haskell, Scheme.