MATH0107 Probabilistic Method in Combinatorics

Year:	2022–2023
Code:	MATH0107
Level:	7 (UG)
Normal student group(s):	UG Year 3 or 4 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0029 or MATH0028, Recommended: MATH0057 or another module in probability
Lecturer:	Dr S Letzter

Course Description and Objectives

The module introduces the probabilistic method, a powerful approach with many applications in combinatorics. The basic idea behind the method is that to prove that a combinatorial object with certain properties exists, it suffices to show that a random construction produces such an object with positive probability. The topics covered in this course will be presented along with applications in several areas in combinatoircs, number theory and geometry.

Recommended Texts

Alon, N. and Spencer, J. H., *The Probabilistic Method*, Wiley.

Detailed Syllabus

- **The basic method.** Ramsey numbers, domination, colouring hypergraphs, set systems.
- **Linearity of expectation.** Maximum cut, sum free sets, Turán's theorem.
- **Alterations.** Large girth and chromatic number, colouring hypergraphs, dependent random choice.
- **The second moment.** Chebyshev's inequality, distinct sums, number of prime divisors, random graphs.
- **Large deviations.** Chernoff bounds, consistent arcs in tournaments.
- **The local lemma.** Proof of the lemma, colouring hypergraphs, colouring real numbers, Ramsey numbers, finding directed cycles.
- **Special topics.** One or more of the following, or related topics.
  - **Correlation inequalities.** FKG inequality, intersecting families.
  - **Martingales.** Azuma–Hoeffding inequality, chromatic number of random graphs, isoperimetric inequality in the hypercube
  - **Extras.** Independence number of triangle-free graphs, crossing numbers.