MATH0104 Modular Forms

Year:	2024–2025
Code:	MATH0104
Level:	7 (UG)
Normal student group(s):	Year 3 and 4 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0013, MATH0014 and MATH0034 (strongly recommended)
Lecturer:	Dr C Busuioc

Course Description and Objectives

This module aims to offer an overview of the basic notions that appear in the classical theory of modular forms. These are analytic objects encoding a lot of arithmetic information which makes them a central point of study in number theory and arithmetic geometry. Modular forms also arise naturally in a variety of other research fields like transcendence proofs, differential equations and mirror symmetry.

The main objects of interest are functions on the complex upper half plane that transform in a special way under the action of $SL_2(\mathbb{Z})$. Two concrete constructions of such functions will be covered: Eisenstein series and theta series. We will also show that the space of modular forms for a specific weight is finite dimensional, which makes all these functions algorithmically computable. Furthermore, we will introduce Hecke operators and show that the L-functions associated to eigenfunctions are of arithmetic nature. Throughout the module, several applications to congruences and positive definite quadratic forms will be highlighted.

Recommended Texts

L.J.P. Kilford, Modular Forms: A Classical and Computational Introduction, Imperial College Press, 2015.
J.-P. Serre, A Course in Arithmetic, Graduate Texts in Mathematics 7, Springer-Verlag, 1973.
W. Stein, Modular Forms, a Computational Approach, Graduate Studies in Mathematics, American Mathematical Society, 2007.
H. Cohen and F. Stromberg, Modular Forms: A Classical Approach, Graduate Studies in Math. 179, American Math. Soc., 2017.

Detailed Syllabus

- The modular group.
- Modular functions and modular forms for the full modular group.
- The dimension of the space of modular forms of weight k.
- Hecke operators.
- The Petersson Inner Product.
- The L-function of a modular form.

- Theta functions.
- Applications to quadratic forms.