

MATH0104 Modular Forms

<i>Year:</i>	2021–2022
<i>Code:</i>	MATH0104
<i>Level:</i>	7 (UG)
<i>Normal student group(s):</i>	Year 3 and 4 Mathematics degrees
<i>Value:</i>	15 credits (= 7.5 ECTS credits)
<i>Term:</i>	2
<i>Assessment:</i>	90% examination, 10% coursework
<i>Normal Pre-requisites:</i>	MATH0013, MATH0014 and MATH0034 (strongly recommended)
<i>Lecturer:</i>	Dr C Busuioac

Course Description and Objectives

This module aims to offer an overview of the basic notions that appear in the classical theory of modular forms. These are analytic objects encoding a lot of arithmetic information which makes them a central point of study in number theory and arithmetic geometry. Modular forms also arise naturally in a variety of other research fields like transcendence proofs, differential equations and mirror symmetry.

The main objects of interest are functions on the complex upper half plane that transform in a special way under the action of $SL_2(\mathbb{Z})$. Two concrete constructions of such functions will be covered: Eisenstein series and theta series. We will also show that the space of modular forms for a specific weight is finite dimensional, which makes all these functions algorithmically computable. Furthermore, we will introduce Hecke operators and show that the L-functions associated to eigenfunctions are of arithmetic nature. Throughout the module, several applications to congruences and positive definite quadratic forms will be highlighted.

Recommended Texts

L.J.P. Kilford, *Modular Forms: A Classical and Computational Introduction*, Imperial College Press, 2015.

J.-P. Serre, *A Course in Arithmetic*, Graduate Texts in Mathematics 7, Springer-Verlag, 1973.

W. Stein, *Modular Forms, a Computational Approach*, Graduate Studies in Mathematics, American Mathematical Society, 2007.

H. Cohen and F. Stromberg, *Modular Forms: A Classical Approach*, Graduate Studies in Math. 179, American Math. Soc., 2017.

Detailed Syllabus

- The modular group.
- Modular functions and modular forms for the full modular group.
- The dimension of the space of modular forms of weight k .
- Hecke operators.
- The Petersson Inner Product.
- The L-function of a modular form.

- Theta functions.
- Applications to quadratic forms.

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