MATH0102 Applied Stochastic Methods

Year:	2024–2025
Code:	MATH0102
Level:	7(UG)/7(PG)
Normal student group(s):	UG Year 3 and 4 Mathematics degrees PG MSc Mathematical Modelling
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	90% examination, 10% coursework
Normal UG Pre-requisites:	MATH0010 and MATH0011 *Useful but not required*: MATH0056, MATH0031, MATH0065, MATH0057
Lecturer:	Prof G Esler

Course Description and Objectives

This module aims to introduce the main analytical techniques and concepts used in the application of stochastic differential equation models to physical, chemical and biological systems.

Recommended Texts

(i) C. W. Gardiner, *Stochastic Methods A Handbook for the Natural and Social Sciences*, Springer, 2009.

(ii) H. Risken, *The Fokker-Planck Equation; Methods of Solution and Applications*, Springer, 1996.

(iii) N. Van Kampen, *Stochastic Processes in Physics and Chemistry*, North Holland, 2007.

(iv) B. Øskandal, *Stochastic Differential Equations. An introduction with applications.*, Springer, 2007.

(v) G. Pavliotis, *Stochastic Processes and Applications: Diffusion Processes, the Fokker-Planck and Langevin Equations.* Springer, 2014.

Detailed Syllabus

– Introduction to applied stochastic methods: Brownian motion and stochastic differential equations. Fokker-Planck and backward Kolmogorov equation. First passage time and exit problems. Feynman-Kac formula and stochastic representations of general linear parabolic and elliptic PDE problems.

– Asymptotic methods: Langevin equation. Adiabatic elimination of fast variables (e.g. Kramers to Smoluchowski equation). Hamiltonian systems perturbed by weak noise. Brownian motion in a potential well, Kramer's method for calculating the escape time. Arrhenius formula. Large-deviation scalings.

– Linear response theory: Linear response problems for SDEs, susceptibility. Fluctuation-dissipation theorem. Green-Kubo formulae.

– Special topics. One or more from:

(a) Freidlin-Wentzel theory of large-deviations. Rare events and instantons.
(b) Stochastic filtering problems.
(c) Stochastic control theory.
(d) Jump processes including Lévy flights.