MATH0092 Variational Methods for Partial Differential Equations

Year:	2021–2022
Code:	MATH0092
Level:	7 (UG)/ 7 (PG)
Normal student group(s):	UG: Year 3 or 4 Mathematics degrees PG: Mathematics students
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0051
Lecturer:	Dr D Hewett

Course Description and Objectives

The aim of this course is to present essential aspects of the modern variational theory of partial differential equations and its application to the analysis of modern numerical methods. It will cover the essential foundations in applied functional analysis and Sobolev spaces for the treatment of weak solutions of partial differential equations. It will also include key aspects of the mathematical analysis of finite dimensional approximations as used in modern numerical computations.

Detailed Syllabus

- Introduction to Hilbert spaces. The Riesz representation theorem for Hilbert spaces, and the Lax–Milgram theorem.
- The Lebesgue spaces $L^p(\Omega)$ on domains $\Omega \in \mathbb{R}^d$, weak derivatives of functions, and Sobolev spaces $W^{k,p}(\Omega)$. Density of smooth functions and approximation theory by piecewise polynomials. Poincaré's inequality in d-dimensions. Embedding theorems, trace theorems.
- Weak formulation of general second-order elliptic partial differential equations. Treatment of boundary conditions. Analysis of coercivity, boundedness, and well-posedness. Regularity theory, and examples of solutions with low regularity.
- Finite dimensional approximations and Galerkin's method. Galerkin orthogonality and Céa's lemma. Application to convergence theory of piecewise-polynomial approximations from the finite element method. Advanced topics in convergence theory and error estimation.
- Introduction to some methods for solving the linear systems associated to finite element approximations.
- Applications to various physical problems from solid and fluid mechanics.