# MATH0090 Elliptic Partial Differential Equations

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| *Year*: | 2021–2022 |
| *Code*: | MATH0090 |
| *Level*: | 7 (UG) |
| *Normal student group(s)*: | UG: Year 3 or 4 Mathematics degrees |
| *Value*: | 15 credits (= 7.5 ECTS credits) |
| *Term*: | 2 |
| *Assessment*: | 90% examination, 10% coursework |
| *Normal Pre-requisites*: | MATH0051, MATH0018, MATH0019<br>(MATH0070 suggested) |
| *Lecturer*: | Dr C Bellettini |

*Course Description and Objectives*

This course is intended as an introduction to the theory of second order elliptic partial differential equations. Elliptic equations play an important role in many fields of geometry and a strong background in linear elliptic equations provides a foundation for understanding other topics such as minimal surfaces, harmonic maps and general relativity. The course is centred around linear theory, with an outlook on non-linear equations. Both classical and weak solutions to elliptic equations are discussed, addressing the existence and uniqueness for solutions to the Dirichlet problem and analysing the regularity of solutions. This involves establishing maximum principles, Schauder estimates (and other estimates on solutions). Finally, we will discuss the De Giorgi-Nash-Moser theory, which can be used for example to establish the regularity of weak solutions to the minimal surface equation (which is non-linear).

*Recommended Texts*

L. C. Evans, *Partial differential equations* D. Gilbarg and N. Trudinger, *Elliptic partial differential equations of second order*

*Detailed Syllabus*

- Introduction: definitions and examples of elliptic PDEs, including some non-linear ones (e.g. minimal surfaces or harmonic maps).
- Weak and strong maximum principle, and their consequences (uniqueness of solutions to the Dirichlet problem).
- Review of Hölder spaces. Interior and boundary Schauder estimates.
- Existence of a solution to the Dirichlet problem: continuity method, Perron's method, barriers.
- Interior and global regularity in $C^{k,a}$-spaces (higher order Schauder estimates).
- Review of Sobolev spaces. Equations in divergence form: variational origin of the equations, weak solutions, existence and regularity theory in $W^{k,2}$-spaces (includes Lax-Milgram theorem, Fredholm alternative).
- De Giorgi-Nash-Moser theory: motivations and statement, some ideas involved in the proof, some applications (e.g. to minimal surfaces).