

MATH0086 Computational and Simulation Methods

<i>Year:</i>	2024–2025
<i>Code:</i>	MATH0086
<i>Level:</i>	7(UG)/7(PG)
<i>Normal student group(s):</i>	UG Year 4 Mathematics degrees PG MSc Mathematical Modelling
<i>Value:</i>	15 credits (= 7.5 ECTS credits)
<i>Term:</i>	1
<i>Assessment:</i>	80% examination and 20% coursework
<i>Normal Pre-requisites:</i>	MATH0056 MATH0058 recommended
<i>Lecturers:</i>	Dr S Timoshin & Prof G van der Heijden
<i>Computer Class Teacher:</i>	TBC

Course Description and Objectives

This module will consider various computational methods which may be used when mathematical modelling. The aim is for the students to investigate the mathematics behind various numerical processes and also the use of software for simulation and visualisation of outcomes. Specific mention will be made of computational experiments which will be linked to issues raised in other components of the Masters course in Mathematical Modelling. The syllabus is broad with the emphasis placed on research applications.

Detailed Syllabus

- **Introduction and ODEs:** Preliminary concepts, including formulation of physical problems from continuum mechanics in terms of ODEs and PDEs. Solution of ODEs by finite difference methods. Euler, Runge-Kutta, linear stability, implicit methods, systems of ODEs, higher order ODEs and the shooting method for boundary problems.
- **PDEs:** Categorization into parabolic, elliptic, hyperbolic types. Examples to include reduction of linear elliptic PDEs to eigenvalue problems using normal mode approach (e.g. plane Poiseuille flow). Direct methods (Gaussian elimination, LU decomposition), iterative techniques (Jacobi and Gauss-Seidel, SOR), vector norms. Finite-differencing for parabolic and hyperbolic PDEs. Explicit, implicit and Crank-Nicholson schemes. Stability criteria.
- **Finite element modelling:** Detailed introduction to finite elements for 1D and 2D problems covering: weak formulation, Galerkin approximation, shape functions, isoparametric elements, application to archetypal second-order (heat equation) and fourth-order (beam equation) problems, eigenvalue problems, nonlinear problems.
- **Computer classes:** Introduction to programming in Matlab; solving problems derived in lectures.