MATH0082 Evolutionary Games and Population Genetics

Year:	2024–2025
Code:	MATH0082
Level:	7 (UG)/ 7(PG)
Normal student group(s):	UG Year 4 Mathematics degrees PG MSc Mathematical Modelling
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0008, MATH0010, and MATH0003 since the course involves proofs of theorems that involve standard concepts such as continuity, limits, extrema, and convexity.
Lecturer:	Prof SA Baigent

Course Description and Objectives

The course concerns two areas of theoretical biology, evolutionary game theory and population genetics where mathematics has played in important role in their theoretical development. While the necessary biological background is covered in the course, the emphasis is more on the mathematical theory. Models are developed and theorems are proved to determine how these models behave under various biological assumptions. Many of the models are dynamical systems, and a significant part of the course is introducing and using methods for differential equations and iterated maps to study them. The generality of the key models enables us to prove a range of theorems that yield important and fundamental results in both mathematical genetics and game theory.

The course may of interest to students from other departments, but they should be aware that it requires a strong mathematical background and a willingness to engage with mathematical proofs.

This course is independent of the course MATH0030 (Mathematical Ecology).

Detailed Syllabus

The topics to be discussed will be chosen from:

(i) *Evolutionary Game Theory*
Fitness pay-offs and symmetric games. Static analysis: Evolutionarily stable strategies and Nash equilibria. Dynamic analysis: Continuous-time replicator dynamics. Lyapunov stability. Asymptotic stability of Evolutionarily stable strategies. Phase portraits. Examples of evolutionary games, including Hawk-Dove, Hawk-Dove-Bully, Rock-Paper-Scissors games. Asymmetric (bimatrix) games. Mathematical description of Mendelian genetics. The Hardy-Weinberg law. Natural selection on gene frequencies (discrete and continuous-time models). Fisher's Fundamental Theorem. Dominance, fixation-time and heterozygote advantage. Mutation-selection balance. Recombination.

(ii) *Mathematical Population Genetics*
Mathematical description of Mendelian genetics. The Hardy-Weinberg law. The Wright-Fisher model for genetic drift, including the Markov chain interpretation and derivation

of the expected time to fixation. Discrete-time models for allele frequencies using maps and their functional compositions. The selection map and its analysis. Mean fitness as a Lyapunov function: Fisher's Fundamental Theorem of Natural Selection. Mutation models. Combined selection-Mutation models. Models for genetic recombination.