MATH0078 Asymptotic Approximation Methods

Year:	2021–2022
Code:	MATH0078
Level:	7(UG)/7(PG)
Normal student group(s):	UG Year 4 Mathematics degrees PG MSc Mathematical Modelling
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0056
Lecturer:	Dr S Timoshin

Course Description and Objectives

This course is on methods for solving problems involving a small parameter. Such problems arise in almost every branch of mathematics and indeed examples of asymptotic (or perturbation) problems appear in many earlier undergraduate courses. The basic idea of asymptotic approximation is to exploit the small parameter to replace the original problem by a sequence of simpler problems providing increasingly accurate approximations. The aim of this course is to present in a systematic manner some powerful approximation techniques typically employed in applied areas, with a focus on finding and interpreting solutions rather than providing rigorous proofs. Three broad classes of asymptotic methods are considered: matched asymptotic expansions, the method of multiple scales, and WKB approximations. The methods are introduced via model problems, such as algebraic and ordinary differential equations, and then applied to problems appearing in applications, which are often expressed in terms of partial differential equations. Examples are taken from a wide range of topics including Fluid Dynamics, Physics, Biomathematics, Financial Mathematics and so on. Prior knowledge in these areas is not necessary as the emphasis in each case is on the asymptotic approximation method rather than the details of the application.

Recommended Texts

Hinch, E. J., *Perturbation Methods*, CUP.
Nayfeh, A.H., *Introduction to Perturbation Techniques*, Wiley.
Kevorkian, J. and Cole, J.D., *Multiple scale and singular perturbation methods*, Springer.
Holmes, M.H., *Introduction to perturbation methods*, Springer.

Detailed Syllabus

- Ideas of asymptotics – small parameters and multiple scales around us.
- Asymptotic expansions – “big O” and “little o” notation, gauge functions, convergence and uniqueness or otherwise.
- Parameter and coordinate expansions. Local similarity and local eigenfunctions.
- Regular vs singular perturbation problems.
- Method of matched asymptotic expansions. Rescaling, distinguished limits. Outer and inner (boundary-layer) expansions. Internal boundary layer. Asymptotic matching using Prandtl’s and Van Dyke’s rules. Matching in the intermediate limit. Uniformly valid expansions.

- Method of multiple scales. Fast and slow variables, solvability conditions.
- WKB method – Eikonal and transport equations. Turning points.
- Examples and applications will be taken from the following: Fluid dynamics at large Reynolds numbers – Blasius boundary layer, wake flow. Falkner-Skan solutions. Ekman and Stewartson layers. Black-Scholes models of options pricing. Reaction-diffusion-advection systems in chemistry and biology. Turing instability. Accretion disks. Quantum tunnelling. Waves in a slowly varying medium. Shoaling and tsunamis. Acoustic waves. Amplitude equations for weakly nonlinear systems. Eigenfunctions with large eigenvalues.