MATH0075 (Lie Groups and Lie Algebras)

Year:	2019–2020
Code:	MATH0075
Old code:	MATHM206
Level:	7 (UG)
Value:	15 units (= 7.5 ECTS credits)
Normal student group(s):	UG Year 4 Mathematics degrees
Term:	1
Structure:	3 hour lectures and 1 hour problem class per week
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0014 (previously MATH2201)
Lecturer:	Prof A Yafaev

Course Description and Objectives

Lie groups are continuous groups of symmetries, like the group of rotations of n-dimensional space or the group of invertible n-by-n matrices. In studying such groups we can use tools from calculus to linearise our problems, which leads us to the notion of a Lie algebra: a vector space with an antisymmetric product associated to any Lie group, which remembers everything about its algebraic structure. For example, the Lie algebra associated with the group of rotations of 3-space is just 3-dimensional Euclidean space with (twice) the vector cross product.

This course divides in two halves. In the first half we introduce the notion of a Lie algebra and the relationship between a Lie group and its Lie algebra. This will involve some ideas from geometry (manifolds and tangent spaces) which will serve you well in later courses. In the second half we study representations of Lie groups and Lie algebras, paying attention to the groups SU(2) and SU(3). This will be much more algebraic. As applications, we will see how the hexagonal wight diagrams of representations of the group SU(3) inspired Gell-Mann and Ne'eman to invent the concept of quarks, and how rotational symmetry of a hydrogen atom helps you understand its orbitals using representation theory. No previous physics knowledge required!

Recommended Texts

(i) R. Carter, G. Segal, I. McDonald, *Lectures on Lie groups and Lie algebras*, LMS Student Texts, 32, CUP 1995.

(ii) W. Fulton, J. Harris, *Representation theory: a first course*, Springer GTM, 129, 1991.

(iii) A. Kirillov, Jr., *An introduction to Lie groups and Lie algebras*, Cambridge Studies in Advanced Mathematics, 113, CUP 2008.

(iv) F. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer GTM, 94,1983.

Detailed Syllabus

1. From Lie groups to Lie algebras

   – Lie groups: definition and examples.

- The adjoint representation.
- The exponential map, Lie bracket and Lie algebras.
- Lie's theorems.

2. Representations

- Representations of Lie groups and Lie algebras.
- Abelian Lie groups.
- Representations of SU(2) and SO(3). The hydrogen atom.
- Representations of SU(3). The eightfold way.