

MATH0074 Topology and Groups

<i>Year:</i>	2024–2025
<i>Code:</i>	MATH0074
<i>Level:</i>	7 (UG)
<i>Normal student group(s):</i>	UG Year 3 and 4 Mathematics degrees
<i>Value:</i>	15 credits (= 7.5 ECTS credits)
<i>Term:</i>	1
<i>Assessment:</i>	90% examination and 10% coursework
<i>Normal Pre-requisites:</i>	MATH0051, MATH0053
<i>Lecturer:</i>	Assoc Prof L Louder

Course Description and Objectives

The course begins with familiar groups and the spaces they act on. The fundamental group is then defined and studied, with an emphasis on examples: graphs and their fundamental groups, free groups, are especially motivating. Fundamental groups of general complexes are discussed, along with group presentations.

Students should finish the course able to compute: bases for the subgroup of a free group corresponding to an explicit covering space of a graph; presentations for fundamental groups of cell complexes; The Wirtinger presentation; Cayley graphs of finite groups.

Recommended Texts

- (i) D. Cohen, *Combinatorial Group Theory: A Topological Approach*, Student Texts 14 (London Mathematical Society, 1989), Chapters 1–7.
- (ii) A. Hatcher, *Algebraic Topology* (CUP, 2001), Chapter. 1.
- (iii) M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chapters. 1–7, 12, 17.
- (iv) D. L. Johnson, *Presentations of Groups*, Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chapters. 1–5, 10, 13.
- (v) W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chapters. 1–4.
- (vi) William S. Massey, *A Basic Course in Algebraic Topology* (Springer-Verlag, 1991), Chapters 2–6.
- (vii) John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

Detailed Syllabus

- Group presentations: Free groups, generators and relations, examples and computations. Cayley graphs.
- Topological spaces; The quotient topology; cell complexes; more examples, including cell and simplicial complexes (recalled from Algebraic Topology). Knot complements.

- Topological spaces, homotopy and the fundamental group. Covering spaces, path lifting and homotopy lifting, covering transformations.
- Applications to computing fundamental groups, e.g. of the circle. Existence and uniqueness of coverings, the universal cover, covers and subgroups, regular coverings and normal subgroups. Pullbacks and intersections. Seifert-Van Kampen theorem.
- Graphs and free groups: reduced words revisited, the Nielsen-Schreier Theorem, computing bases for subgroups of free groups.
- Free products, free products with amalgamation, groups acting on trees, Kurosh subgroup theorem, Grushko's theorem.
- The Freiheitssatz