# MATH0074 Topology and Groups

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| *Year:* | 2021–2022 |
| *Code*: | MATH0074 |
| *Level:* | 7 (UG) |
| *Normal student group(s):* | UG Year 3 and 4 Mathematics degrees |
| *Value*: | 15 credits (= 7.5 ECTS credits) |
| *Term*: | 1 |
| *Assessment*: | 90% examination and 10% coursework |
| *Normal Pre-requisites*: | MATH0051, MATH0053, and MATH0052 recommended |
| *Lecturer*: | Assoc Prof L Louder |

*Course Description and Objectives*

The course begins with familiar groups and the spaces they act on. The fundamental group is then defined and studied, with an emphasis on examples: graphs and their fundamental groups, free groups, are especially motivating. Fundamental groups of general complexes are discussed, along with group presentations.

Students should finish the course able to compute: bases for the subgroup of a free group corresponding to an explicit covering space of a graph; presentations for fundamental groups of cell complexes; The Wirtinger presentation; Cayley graphs of finite groups.

*Recommended Texts*

(i) D. Cohen, *Combinatorial Group Theory: A Topological Approach*, Student Texts 14 (London Mathematical Society, 1989), Chapters 1–7.

(ii) A. Hatcher, *Algebraic Topology* (CUP, 2001), Chapter. 1.

(iii) M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chapters. 1–7, 12, 17.

(iv) D. L. Johnson, *Presentations of Groups*, Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chapters. 1–5, 10, 13.

(v) W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chapters. 1–4.

(vi) William S. Massey, *A Basic Course in Algebraic Topology* (Springer-Verlag, 1991), Chapters 2–6.

(vii) John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

*Detailed Syllabus*

– Group presentations: Free groups, generators and relations, examples and computations. Cayley graphs.

– Topological spaces; The quotient topology; cell complexes; more examples, including cell and simplicial complexes (recalled from Algebraic Topology). Knot complements.

– Topological spaces, homotopy and the fundamental group. Covering spaces, path lifting and homotopy lifting, covering transformations.

– Applications to computing fundamental groups, e.g. of the circle. Existence and uniqueness of coverings, the universal cover, covers and subgroups, regular coverings and normal subgroups. Pullbacks and intersections. Seifert-Van Kampen theorem.

– Graphs and free groups: reduced words revisited, the Nielsen-Schreier Theorem, computing bases for subgroups of free groups.

– Free products, free products with amalgamation, groups acting on trees, Kurosh subgroup theorem, Grushko's theorem.

– The Freiheitssatz