# MATH0073 Representation Theory

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| *Year:* | 2024–2025 |
| *Code*: | MATH0073 |
| *Level:* | 7 (UG) |
| *Normal student group(s):* | UG: Year 3 and 4 Mathematics degrees |
| *Value*: | 15 credits (= 7.5 ECTS credits) |
| *Term*: | 2 |
| *Assessment*: | 60% Final Exam, 30% Mid-term Test, 10% Coursework |
| *Normal Pre-requisites*: | MATH0053 |
| *Lecturer*: | Dr E Segal |

*Course Description and Objectives*

A representation of a group is a way of realising it as a group of invertible matrices, or – more abstractly – as a group of symmetries of a vector space. The theory of representations thus involves a rich interplay of group theory and linear algebra. It has many applications in theoretical physics, as well as being a building block for more advanced topics in algebra.
This course will mostly study the representations of finite groups over the field of complex numbers. After some basic definitions and examples we will develop some general theory, including the foundational Maschke's Theorem and Schur's Lemma, which will allow us to completely classify the representations of many groups. We'll then introduce the powerful theory of characters, which reduces many questions in representation theory to simple numerical calculations. In the final section of the course we'll introduce group algebras and see how our results fit into the more general theory of modules.

*Detailed Syllabus*

First properties and examples of representations. Permutation representations. The regular representation. Geometric constructions of representations. Subrepresentations, irreps. G-linear maps. Maschke's Theorem. Schur's lemma. Representations of abelian groups. Duals and tensor products.

Characters. Inner products. Character tables, row and column orthogonality.

Group algebras. Representations as modules over the group algebra. Isomorphism of the group algebra with a product of matrix algebras.

If time permits:
Classification of semi-simple algebras.
Induced and co-induced representations, Frobenius Reciprocity.
Basics of representation theory over the reals and finite fields.
The Eckmann-Shapiro Lemma and Burnside's Theorem.