MATH0072 Riemannian Geometry

Year:	2023–2024
Code:	MATH0072
Level:	7 (UG)
Normal student group(s):	UG Year 4 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0019, MATH0020
Lecturer:	Prof M Singer

Course Description and Objectives

Differential and Riemannian Geometry provide an important tool in modern mathematics, impacting on diverse areas from the pure to the applied. The first aim of this course is to give a thorough introduction to the theory of abstract manifolds, which are the fundamental objects in Differential Geometry. The second aim is to describe the basics of Riemannian Geometry, in particular the notion of geodesics and curvature. Our final objective will be to analyse manifolds with constant curvature, with a focus on the sphere and hyperbolic space. By the end of the course, students will have a thorough understanding of curved spaces and will have gained an introduction to the applications of Riemannian Geometry to topology.

Recommended Texts

(i) W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, Chapters 1–5, 7–8.

(ii) S. Gallot, D. Hulin and J. Lafontaine, *Riemannian Geometry*, Chapters I-III.

(iii) M. do Carmo, *Riemannian Geometry*, Chapters 0–4, 7–8.

(iv) J. M. Lee, *Riemannian Manifolds: An Introduction to Curvature.*

(v) J. Jost, *Riemannian Geometry and Geometric Analysis*, Chapters 1, 4–5.

Detailed Syllabus

Definition of abstract manifolds and examples, smooth maps. Tangent vectors and the tangent bundle, push-forward, vector fields and flows, Lie bracket. Differential forms, exterior derivative, interior and wedge product, pull-back. Tensor fields, tensor product. Lie derivative. Partition of unity, orientation, Riemannian metric, Cartan's formula.

Definition of Riemannian manifolds and examples. Levi-Civita connection, parallel transport. Geodesics, exponential map, curvature and examples. Completeness and Hopf–Rinow Theorem. Manifolds with constant curvature, sphere, geometry of hyperbolic space, classification.

Further topics will be selected from: Jacobi fields, Riemannian submanifolds, applications of curvature to topology (including the Cartan–Hadamard Theorem, Bonnet–Myers Theorem, Synge–Weinstein Theorem and the Sphere Theorem), applications to geometric analysis (including the Laplace–Beltrami operator, the Hodge Laplacian, harmonic forms, Hodge's Theorem).