MATH0070 (Linear Partial Differential Equations)

Year:	2019–2020
Code:	MATH0070
Old code:	MATHM110
Level:	7(UG)
Normal student group(s):	Year 3 or 4 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Structure:	3 hours lectures per week. Weekly assessed coursework.
Assessment:	The final weighted mark for the module is given by: 90% examination, 10% coursework.
Normal Pre-requisites:	MATH0013 (previously MATH2101) MATH0016 (previously MATH2401) MATH0051(previously MATH7102)
Lecturer:	Dr I Kamotski

Course Description and Objectives

Partial differential equations are important in many fields of mathematics and are the essential language of physical applied mathematics, where they are used to model phenomena including wave propagation, heat flow, as well as soap films and soap bubbles. This course provides an introduction to some of the mathematical techniques needed to study linear partial differential equations and serves as a foundation for more advanced work on nonlinear PDE and PDE on manifolds. Tools such as the theory of distributions and the Fourier transform are of wide applicability beyond the theory of PDEs and are of great interest in their own right.

The objectives of the course are to introduce test functions (smooth functions with compact support) and distributions, the Schwartz space, and then study the Fourier transform in the Schwartz space and L^2. Using these tools we shall then be able to write down fundamental solutions for a large class of linear differential operators and will be able to study the qualitative differences between elliptic, parabolic and hyperbolic partial differential equations. The Fourier transform gives access to the simplest L^2-based Sobolev spaces and allows us to give basic versions of elliptic regularity for constant-coefficient operators. The course will conclude with a detailed study of harmonic functions, emphasising the parallels with complex function theory and will use Perron's method to study the Dirichlet problem for domains in R^n.

Recommended Texts

1. Friedlander and Joshi, 'Introduction to the theory of distributions', Chapters 1-9

2. Gilbarg and Trudinger, 'Elliptic partial differential equations of second order', Ch 2 (for the last section on harmonic functions)

3. L. C. Evans, 'Partial differential equations' Part I, Chapter 2

Detailed Syllabus

- Basics on smooth functions and test functions.
- Distributions—definitions, differentiation, multiplication by smooth functions, etc.
- Linear differential operators: elliptic/parabolic/hyperbolic classification.
- Distributions with compact support, distributions supported at a point.
- Convolution and fundamental solutions of linear PDEs; the classical integral representation formulae. Statement of Schwartz kernel theorem.
- Fourier transform, convolution, Poisson's summation formula, Sobolev spaces, elliptic regularity.
- Maximum principles and energy.
- The Dirichlet problem for harmonic functions.