MATH0069 Probability

Year:	2024–2025
Code:	MATH0069
Level:	7(UG)/7(PG)
Normal student group(s):	UG Year 3 or 4 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0017
Lecturer:	Prof N Sidorova

Course Description and Objectives

This course follows on from earlier courses in real analysis and measure theory, and describes what is perhaps the most important application of measure theory in mathematics; the rigorous theory of probability. The course material is focused on the two most fundamental principles of the theory: the strong law of large numbers and the central limit theorem.

Recommended Texts

A recommended text is D Williams, *Probability with Martingales*.

Detailed Syllabus

Probability space, events, random variables. Probability laws, distribution function, densities. Joint laws. Expectation, variance. Chebyshev's inequality.

Independence. The Borel-Cantelli Lemmas. Bernstein's inequality. Strong law of large numbers for i.i.d. Bernoulli random variables. Tail σ-algebras and Kolmogorov's $0-1$ law.

Weak convergence. Characteristic functions and Fourier transforms. The Parseval-Plancherel Theorem. The Central Limit Theorem.

Conditional expectations, filtrations, and martingales. Stopping times. The Optional Stopping Theorem with applications. Doob's inequality. Kolmogorov's inequality. Kolmogorov's Theorem about the series. Strong Law of Large Numbers.

Previsible processes and the martingale transform. Doob's Uncrossing Lemma. Martingale Convergence Theorem.