MATH0065 (Advanced Modelling Mathematical Techniques)

Year:	2020–2021
Code:	MATH0065
Level:	7(UG)/7(PG)
Normal student group(s):	UG Year 4 Mathematics degrees PG MSc Mathematical Modelling
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Structure:	Online
Assessment:	100% examination
Normal UG Pre-requisites:	MATH0010, MATH0011, and MATH0013 Some knowledge of 2D fluid mechanics useful, but not essential.
Lecturers:	Dr S Timoshin and Prof G Esler

Course Description and Objectives

This module aims to ensure that students possess knowledge of the analytical techniques used in mathematical modelling.

Recommended Texts

(i) Hinch, *Perturbation methods*, Cambridge Texts in Applied Mathematics.

(ii) Kevorkian & Cole, *Perturbation methods in applied mathematics*, Applied Mathematical Sciences, Springer.

(iii) Bender & Orszag, *Advanced mathematical methods for scientists and engineers*, Asymptotic Methods and Perturbation Theory: v.1, Springer.

(iv) Churchill and Brown, *Complex variables and applications*, McGraw-Hill.

(v) Nehari, *Introduction to complex analysis*, Allyn and Bacon.

(vi) G. Pavliotis, *Stochastic Processes and Applications: Diffusion Processes, the Fokker-Planck and Langevin Equations.* Springer, 2014.

Detailed Syllabus

– Partial differential equations (PDEs): Revision of main solution techniques for elliptic and parabolic PDEs (separation of variables, Fourier transforms, similarity variables, Green's functions).

– Introduction to applied stochastic methods: Brownian motion and stochastic differential equations. Connection to PDEs: Fokker-Planck and backward Kolmogorov equation. Exit time problems.

– Perturbation Methods. Introduction to modelling concepts, dimensional analysis, perturbation techniques, matched asymptotics.

– Application of Complex Variables. Conformal mapping and applications. A selection from:

  (a) Hodograph and potential-plane techniques
  (b) Schwartz functions and vortex equilibria
  (c) Hele-Shaw free boundary problems
  (d) Two-dimensional freezing/melting problems