

MATH0056 Mathematical Methods 4

<i>Year:</i>	2023–2024
<i>Code:</i>	MATH0056
<i>Level:</i>	6 and 7(UG)
<i>Normal student group(s):</i>	UG Year 2 and 3 Mathematics degrees (level 6); Year 4 Engineering degrees (level 7)
<i>Value:</i>	15 credits (= 7.5 ECTS credits)
<i>Term:</i>	2
<i>Assessment:</i>	90% examination, 10% coursework
<i>Normal Pre-requisites:</i>	MATH0013, MATH0016
<i>Lecturer:</i>	Prof G Esler

Course Description and Objectives

This course continues from the course MATH0016 and aims to introduce further tools required to solve the partial differential equations which arise in applied mathematics. It first looks at the application of the separation of variables method in cylindrical and spherical coordinates. This necessitates a study of Bessel's and Legendre's equations and their solutions. This is done via a combination of the Frobenius method of series solution of ODE's together with generating functions.

The course then moves on to study transform methods of solving PDEs, complementing the method of separation of variables and concentrating on the Fourier and Laplace transforms. The necessary techniques of integration in the complex plane will be reviewed.

Recommended Texts

Relevant books are: (i) *Advanced Mathematical Methods for Engineering & Science Students*, G Stephenson & P M Radmore; (ii) (more advanced) *Applied Complex Variables*, J W Dettman (Dover).

Detailed Syllabus

The course may be described under the following headings:

- Laplace's equation in cylindrical coordinates and its solution by the separation of variables. Legendre's equation and its series solution. Legendre Polynomials, P_n , their orthogonality, and their generating function. Applications of the generating function.
- The wave equation in cylindrical coordinates and its solution by separation of variables. Bessel's equation. The general Frobenius method and its application to Bessels equation. J_n , Y_0 . The generating function for J_n and its application. Fourier-Bessel series and normal modes of oscillation.
- Laplace and Fourier transforms. Their definition and inversion using tables in conjunction with the shift and dilation theorems and the convolution theorem. Inversions using complex integration, including integration around branch cuts. Their use in the solutions of ODEs and PDEs and integral equations of the convolution type.