MATH0055 Mathematics of Electromagnetism and Special Relativity

Year:	2024–2025
Code:	MATH0055
Level:	5 (UG)
Normal student group(s):	UG: Year 2 and 3 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	The final weighted mark for the module is given by: 90% examination, 10% coursework
Normal Pre-requisites:	MATH0016
Lecturer:	Dr M Kwasigroch

Course Description and Objectives

'"Let there be light," and there was light.' Maxwell's equations are a set of partial differential equations which describe light and many other phenomena related to electromagnetism. Many optical, electrical, and radio technologies are covered by the Maxwell equations. This includes simple examples like the electrical field around a wire or modern applications like bluetooth wireless technology. Perhaps surprisingly, the Maxwell equations contain the initial seeds of Einstein's theory of special relativity which is required for a functioning GPS tracking system, for example. They also contain the seeds of quantum mechanics.
The course motivates Maxwell's equations from the wave equation and continues with simple applications of the theory, following by establishing solutions which can describe the propagation of electromagnetic waves, i.e. light! The final part of the course develops Einstein's special relativity and derives the wonderful equation $E = mc^2$.
The course aims to provide students who have an interest in mathematical physics with an introduction to classical electromagnetism and relativistic mechanics. The course should also be of interest to students wishing to see further application of the ideas covered in mathematical methods courses.

By the end of this course students should have

- An understanding of steady and time-varying electric and magnetic fields and their description through Maxwell's equations, both in integral and differential form and scalar and vector potentials.
- The ability to calculate steady solutions to these equations for simple geometries and as far-field expansions for more general situations. The ability to calculate electrostatic and magnetic energy, capacitance and inductance for simple geometries.
- An understanding of electromagnetic wave propagation in a vacuum and of energy and momentum flow within time-varying fields and a description of the fields in terms of retarded potentials.
- An understanding of special theory of relativity, space-time, relativistic mechanics and the behaviour of magnetic and electric fields under Lorentz transformation.

Recommended Texts

- Introduction to Electrodynamics, D.J. Griffiths, ISBN: 9780138053260, Pearson.
- The Feynman Lectures on Physics - Volume II, R.P. Feynman, R.B. Leighton, M.L. Sands, and M.A. Gottlieb, ISBN: 9780805390476, Pearson/Addison-Wesley.
- Special Relativity, N.M.J Woodhouse, ISBN: 1852334266, Springer Undergraduate Mathematics Series.
- Electricity and Magnetism, W.N. Cottingham and D.A. Greenwood, ISBN: 9780521368032, Cambridge University Press

Detailed Syllabus

- Charges, currents and fields. Wave equation in three dimensions. Green's function for the wave equation. Superposition. Contravariant and covariant vectors. Charge conservation. Gauge freedom. Electric and magnetic fields. Lack of magnetic monopoles. Maxwell's equations and their integral forms.
- Electrostatics. Gauss' theorem. Coulomb force. Electric Potential. Green's functions for the Laplace equation. The steady electric field for discrete and continuous distribution of charge. Multipole expansions. Conductors. Surface charge. Boundary conditions at a surface. Energy. Capacitance.
- Electric Currents. Magnetostatics. The Coulomb Gauge. Magnetic Potential. Biot-Savart Law. Boundary conditions at a surface. Ampere's Law. Electromagnetic Induction. Lorentz force. Magnetic force on conductors. The displacement current. Self-inductance. Magnetic Energy. Relaxation of a charge distribution within a conductor.
- Electromagnetic waves. Energy and momentum transport in an electromagnetic field. The Poynting vector. The Lorenz Gauge. Wave equations for the electric and magnetic potential. Retarded time. Radiation and instability of classical atoms.
- Special relativity. Frame invariance. Tensors and metrics. Invariance of $dx^2 - c^2dt^2$. Lorentz transformations, transformation of velocities. Proper time. Relativistic mechanics. Equations of electromagnetism in space-time.